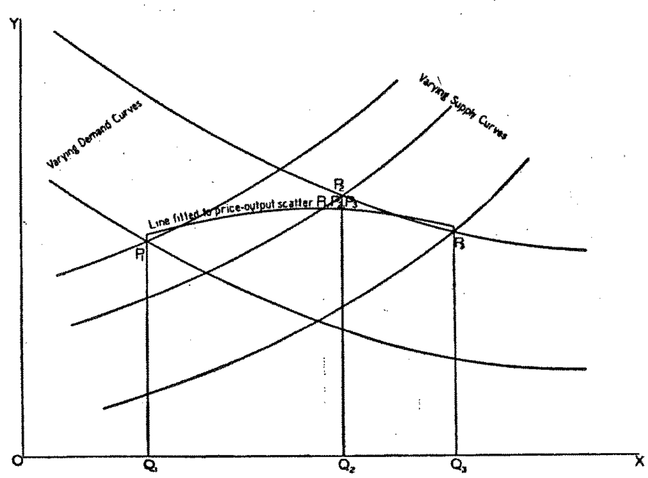
Simultaneous Equations Models

1 All Together Now

- Some types of policy analysis require we know the elasticity (slope) of demand and/or supply. But, which comes first, price or quantity? In *equilibrium*, the direction of causality is unclear
- A regression of quantity on price gives the best (MMSE) approximation to $E[q_t \mid p_t]$ for equilibrium quantities and prices. The figure below suggests that the regression of quantity on price characterizes neither the supply nor demand function, but rather something in between:

If both supply and demand conditions change, price-output data yield no direct information as to either curve. (Figure 4.)

FIGURE 4. PRICE-OUTPUT DATA FAIL TO REVEAL EITHER SUPPLY OR DEMAND CURVE.



- This figure comes from a 1928 study by economist and poet Phillip Wright called *The Tariff on Animal and Vegetable Oils*. Little-noticed at the time, Wright's "Appendix B" laid the intellectual foundations of modern econometrics.
- Wright's problem is not *statistical*. Rather, much as in the story of short and long regressions, the regression we've got is not the one we want. Except that, here, the "regression we want" is not a regression at all, but rather a theoretical economic relationship that describes the counterfactual choices of buyers and sellers.
  - Many economists refer to theoretical relationships of the sort described by supply and demand curves as "structural"
  - Applied econometricians interested in uncovering structural relationships are said to face an *identification problem*

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1.1 Simultaneous Equations Bias

- When prices and quantities are determined by solving two (or more) equations simultaneously, OLS estimates are inconsistent for supply and demand elasticities; this bad behavior is called *simultaneous equations bias*

- As always, we use economic models to understand, simplify, and solve problems. Our problem-solving model here asserts that potential quantities supplied and demanded can be written as a linear function of prices and possibly other variables that we think of as shifting these functions. Specifically, we write:

$$q_t^d(p) = \alpha_0 + \alpha_1 p + \alpha_2 z_t + \epsilon_t^d \quad (1)$$
$$q_t^s(p) = \beta_0 + \beta_1 p + \beta_2 x_t + \epsilon_t^s, \quad (2)$$

  where $q_t^d(p)$ is the quantity consumers demand at price p, $q_t^s(p)$ is the quantity producers supply at price p, and z_t and x_t are additional observed determinants of demand and supply (e.g., income, other prices). The market equilibrium p_t solves

$$q_t^d(p_t) = q_t^s(p_t) = q_t \quad (3)$$

- Variables determined jointly by solving the system (in this case by an equilibrium condition) are said to be *endogenous*. Variables like z_t and x_t, determined outside the system, are said to be *exogenous*.

- What do OLS estimates of (1) or (2) produce when the observed p_t and q_t satisfy equations (1)-(3)?

  – First solve for the *reduced form* for p_t by equating supply and demand:

$$\begin{aligned} p_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} - \frac{\alpha_2}{\alpha_1 - \beta_1} z_t + \frac{\beta_2}{\alpha_1 - \beta_1} x_t + \frac{\epsilon_t^s - \epsilon_t^d}{\alpha_1 - \beta_1} \\ &= \pi_{10} + \pi_{11} z_t + \pi_{12} x_t + \nu_{1t} \end{aligned} \quad (4)$$

  Note that the random part of p_t – the error term ν_{1t} – is surely correlated with both structural errors

- Moral: The supply and demand equations in a simultaneous equations model are not regressions; OLS does not reliably estimate them

- Old-school SEMs are not much seen in modern empirical work, but Wright's analysis of the SEM is of enormous intellectual importance. The SEM remains the foundation upon which the modern 'metrics house is built. Encountering Wright's elegant framework as a college sophomore in 1980, I was floored.

2 The Structure and the Reduced Form: Identification

- The reduced form for p_t is (4). The reduced form for q_t is shown below:

$$\begin{aligned} q_t &= \frac{\beta_1 \alpha_0 - \alpha_1 \beta_0}{\beta_1 - \alpha_1} + \frac{\alpha_2 \beta_1}{\beta_1 - \alpha_1} z_t - \frac{\alpha_1 \beta_2}{\beta_1 - \alpha_1} x_t + \frac{\beta_1 \epsilon_t^s - \alpha_1 \epsilon_t^d}{\beta_1 - \alpha_1} \\ &= \pi_{20} + \pi_{21} z_t + \pi_{22} x_t + \nu_{2t} \end{aligned} \quad (5)$$

- *Reduced form equations are regressions*: their errors are uncorrelated with RHS variables. This is because regressors in the reduced form (x_t and z_t) are assumed to be uncorrelated with the structural errors. In general, reduced form equations–one for each endogenous variable–are found by solving for all endogenous variables as a function of the exogenous variables in the system

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- *When is an SEM identified?* When the structural coefficients can be obtained from the reduced form.
  - You can easily solve for the structural coefficients in the case of equations (4) and (5); try this and see
  - But ponder this riff on (1) and (2):

$$q_t^d(p) = \alpha_0 + \alpha_1 p + \alpha_2 z_t + \alpha_3 x_t + \epsilon_t^d \tag{6}$$
$$q_t^s(p) = \beta_0 + \beta_1 p + \beta_2 z_t + \beta_3 x_t + \epsilon_t^s \tag{7}$$

    This system is *under-identified*, knowledge of the reduced form does not reveal the structure (show this too)

- Identification in the SEM requires *exclusion restrictions*; verily, identification requires instruments! (as we will soon see)
- **Heads-up on the lingo**:
  - The term *reduced form*, which we've been using since Day One, originates in early formal analyses of the SEM
  - In an SEM context, the *reduced form* for price is also the *first stage* for price when we do 2SLS for a demand or supply equation. In the SEM, there's a reduced for for each endogenous variable, including the endogenous variables found on the right-hand side.

Structural vs Reduced-Form Policy Analysis

- Structural equations are models for potential outcomes: they tell us what would happen under alternative random assignments of right-hand-side variables. Read all about it in Angrist, Graddy, and Imbens (2000).
- As we've seen, some policy questions can be tackled directly: Card and Krueger (1994) estimate minimum wage effects without benefit of a structural model. Theirs is sometimes said to be a "reduced form policy analysis" because we can think of the minimum wage as an implicit instrument for the price of labor.
- Supply and demand elasticities are often of intrinsic interest, however, as in the Graddy (1995) study of price discrimination. Do Asians pay less for fish because their demand for fish is more elastic? We require structural demand elasticities to see.
- Some reduced-form questions require extrapolation beyond the data at hand. Consider for example, the possible consequences of a heretofore-unseen $20 minimum wage. Structural parameters can be used to predict the consequences of this.

3 Estimating Simultaneous Equations Models

- OLS on the structural equations doesn't work. What does?
- Indirect least squares (ILS), instrumental variables (IV), and two-stage least squares (2SLS)

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3.1 Indirect Least Squares

- We consistently estimate reduced form coefficients (the π's, above) by OLS because reduced form errors are uncorrelated with exogenous variables
- Indirect least squares (ILS) solves for the structural coefficients from the reduced form estimates ... *if the model is identified*. Using (4) and (5), we find:

$$\pi_{11} = \frac{-\alpha_2}{\beta_1 - \alpha_1} \qquad \pi_{21} = \frac{-\beta_1\alpha_2}{\beta_1 - \alpha_1}$$
$$\pi_{12} = \frac{\beta_2}{\beta_1 - \alpha_1} \qquad \pi_{22} = \frac{\beta_2\alpha_1}{\beta_1 - \alpha_1},$$

so that

$$\frac{\pi_{21}}{\pi_{11}} = \beta_1 \qquad \frac{\pi_{22}}{\pi_{12}} = \alpha_1$$

We can also solve for the structural coefficients on exogenous variables as:

$$-\pi_{12}(\beta_1 - \alpha_1) = \beta_2 \qquad -\pi_{11}(\beta_1 - \alpha_1) = \alpha_2$$

- Substituting sample analogs for population π's in the above formulas, the resulting structural coefficient estimates are ILS estimates.
  - ILS estimates of structural parameters are consistent (why?)

3.2 Two-Stage Least Squares (2SLS) ... Again

- 2SLS logic: substitute first stage fitted values for RHS endogenous variables
  - Consider a simple system with one exogenous variable

$$q_t^d = \alpha_0 + \alpha_1 p_t + \epsilon_t^d \tag{8}$$
$$q_t^s = \beta_0 + \beta_1 p_t + \beta_2 x_t + \epsilon_t^s \tag{9}$$

The reduced form for price is

$$\begin{aligned} p_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} x_t + \frac{\epsilon_t^s - \epsilon_t^d}{\alpha_1 - \beta_1} \\ &= \pi_{10} + \pi_{11} x_t + \nu_{1t} \end{aligned} \tag{10}$$

- Use equation (8) to write

$$q_t = \alpha_0 + \alpha_1 \hat{p}_t + [\epsilon_t^d + (p_t - \hat{p}_t)\alpha_1], \tag{11}$$

where

$$\hat{p}_t = \hat{\pi}_{10} + \hat{\pi}_{11} x_t$$

- Note that $\hat{p}_t$ is a linear function of x_t, which is uncorrelated with ϵ_t^d. Also, $\hat{p}_t$ is necessarily uncorrelated with $(p_t - \hat{p}_t)$. Thus, OLS estimates of α_1 in (11) are consistent
  - OLS regression on fitted values from a first stage is the *2SLS second stage*
- In a simple model with one instrument and one endogenous variables, like (11), 2SLS estimates are identical to the corresponding ILS and IV estimates

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3.3 Over-Identified Models

- Up your game by adding an extra instrument to supply:

$$q_t^d(p) = \alpha_0 + \alpha_1 p_t + \epsilon_t^d \tag{12}$$

$$q_t^s(p) = \beta_0 + \beta_1 p_t + \beta_2 x_t + \beta_3 z_t + \epsilon_t^s \tag{13}$$

  The demand equation is now *over-identified,* meaning we have more than one ILS solution for the identified demand slope, α_1, and more than one instrument available for IV and 2SLS estimation

  - To see this, note that the reduced forms for this system are

$$\begin{aligned} p_t &= \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{\beta_2}{\alpha_1 - \beta_1} x_t + \frac{\beta_3}{\alpha_1 - \beta_1} z_t - \frac{\epsilon_t^s - \epsilon_t^d}{\alpha_1 - \beta_1} \\ &= \pi_{10} + \pi_{11} x_t + \pi_{12} z_t + \nu_{1t} \\ q_t &= \frac{\beta_1 \alpha_0 - \alpha_1 \beta_0}{\beta_1 - \alpha_1} - \frac{\alpha_1 \beta_2}{\beta_1 - \alpha_1} x_t - \frac{\alpha_1 \beta_3}{\beta_1 - \alpha_1} z_t + \frac{\beta_1 \epsilon_t^s - \alpha_1 \epsilon_t^d}{\beta_1 - \alpha_1} \\ &= \pi_{20} + \pi_{21} x_t + \pi_{22} z_t + \nu_{2t} \end{aligned}$$

    These reduced forms generate two solutions for α_1:

$$\alpha_1 = \frac{\pi_{21}}{\pi_{11}} \text{ or } \alpha_1 = \frac{\pi_{22}}{\pi_{12}}$$

- We also have two instruments available to estimate the demand slope:

$$\alpha_1 = \frac{\pi_{21}}{\pi_{11}} = \frac{C(q_t,\, x_t)}{C(p_t,\, x_t)} = \frac{\pi_{22}}{\pi_{12}} = \frac{C(q_t,\, z_t)}{C(p_t,\, z_t)}$$

- 2SLS is an efficient way to combine these:

  - First, compute

$$\begin{aligned} p_t &= \pi_{10} + \pi_{11} x_t + \pi_{12} z_t + \nu_{1t} \\ \hat{p}_t &= \hat{\pi}_{10} + \hat{\pi}_{11} x_t + \hat{\pi}_{12} z_t \end{aligned}$$

    using OLS

  - The 2SLS estimate of α_1 is then the sample analog of

$$\frac{C(q_t,\, \hat{p}_t)}{C(p_t,\, \hat{p}_t)} = IV \text{ using } \hat{p}_t \text{ as an instrument } = \frac{C(q_t,\, \hat{p}_t)}{V(\hat{p}_t)} = OLS \text{ slope for } q_t \text{ on } \hat{p}_t$$

- Note that $\hat{p}_t$ is a linear combination of the two instruments, x_t and z_t

  - In fact, 2SLS is the efficient IV estimator for a homoscedastic over-identified model. That is, the first-stage fitted value is the best single instrument constructed from linear combos of the multiple instruments at hand.

- This rule is unchanged: in practice, when *doing* the SEM, we do 2SLS!

  - 2SLS estimates of ovwr-identified models are statistically efficient, but 2SLS estimates of an under-identified models do not exist; test your SEM understanding by showing this second point

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4 Something Fishy at the Fulton Fish Market

- Graddy, (1995) and Angrist, Imbens, Graddy (1995) estimate the elasticity of demand for fish using this model

$$q_t^d(p_t) = \alpha_0 + \alpha_1 p_t + \alpha_2 x_t + \epsilon_t^d \quad (14)$$
$$q_t^s(p_t) = \beta_0 + \beta_1 p_t + \beta_2 x_t + \beta_3 z_t + \epsilon_t^s \quad (15)$$
$$q_t^d = q_t^s = q_t$$

- Price reduced form:

$$p_t = \pi_{10} + \pi_{11} x_t + \pi_{12} z_t + \nu_{1t}$$

where

q_t =daily quantity demanded from a dealer at Fulton
p_t =average daily price
x_t =vector of day-of-the-week dummies
z_t =vector of measures of weather conditions at sea (wind speed and wave height)

- Annoying details
  1. Data available for only one dealer
  2. No information available on prices of substitute goods

```
. sum

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
     price_a |        97    .8198457    .3518089   .2666667   1.928571
     price_w |        97    .8919645    .3351612        .25   1.708333
       qty_a |        97    2589.763    1904.363        110       9120
       qty_w |        97    1537.454    1199.444         60       6800
        day1 |        97     .185567    .3907764          0          1
-------------+--------------------------------------------------------
        day2 |        97    .1958763    .3989354          0          1
        day3 |        97    .2061856    .4066669          0          1
        day4 |        97    .2061856    .4066669          0          1
      speed2 |        97    11.97938    3.944879          5         25
       wave2 |        97    5.092784    1.787296        2.5       12.5
-------------+--------------------------------------------------------
      speed3 |        97    20.92784    6.524389         10         45
       wave3 |        97    5.020619    1.914742          3       12.5
    avgprice |        97    .8473869     .344057   .2902674   1.775487
    totalqty |        97    4127.216    2620.681        170      10940
```


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```
. gen mixed3=(1-stormy3)*(speed3>15)*(wave3>3)

. gen stormy2=(speed2>12)*(wave2>5.5)

. gen mixed2=(1-stormy2)*(speed2>10)*(wave2>3)

.
. // OLS and reduced form estimates
. reg lnqty day1 day2 day3 day4 lnprice

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  5,    91) =    5.04
       Model |  12.1722085     5  2.4344417            Prob > F      =  0.0004
    Residual |   43.960225    91  .483079395           R-squared     =  0.2168
-------------+------------------------------           Adj R-squared =  0.1738
       Total |  56.1324335    96  .584712849           Root MSE      =  .69504

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        day1 |  -.3109272   .2258233    -1.38   0.172    -.7594975    .1376431
        day2 |  -.6827901    .222667    -3.07   0.003    -1.125091   -.2404896
        day3 |  -.5338939   .2199374    -2.43   0.017    -.9707725   -.0970152
        day4 |   .0672273   .2204205     0.30   0.761     -.370611    .5050656
     lnprice |  -.5246553   .1761115    -2.98   0.004    -.8744792   -.1748314
       _cons |   8.244317   .1628134    50.64   0.000     7.920909    8.567726
------------------------------------------------------------------------------

. reg lnqty day1 day2 day3 day4 speed3

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  5,    91) =    4.77
       Model |  11.6510004     5  2.33020008           Prob > F      =  0.0006
    Residual |  44.4814331    91  .488806957           R-squared     =  0.2076
-------------+------------------------------           Adj R-squared =  0.1640
       Total |  56.1324335    96  .584712849           Root MSE      =  .69915

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        day1 |  -.2669628   .2278686    -1.17   0.244    -.7195958    .1856702
        day2 |  -.5323006   .2301553    -2.31   0.023    -.9894759   -.0751253
        day3 |  -.4803295   .2228455    -2.16   0.034    -.9229846   -.0376743
        day4 |   .0049691   .2211368     0.02   0.982     -.434292    .4442301
      speed3 |  -.0316299   .0113951    -2.78   0.007    -.0542647    -.008995
       _cons |   8.999321    .269828    33.35   0.000     8.463341    9.535302
------------------------------------------------------------------------------

. reg lnprice day1 day2 day3 day4 speed3

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  5,    91) =    2.17
       Model |   1.6717106     5   .33434212           Prob > F      =  0.0646
    Residual |  14.0413996    91  .154301095           R-squared     =  0.1064
-------------+------------------------------           Adj R-squared =  0.0573
```


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```
       Total |  15.7131102    96  .163678231           Root MSE      =  .39281

------------------------------------------------------------------------------
     lnprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        day1 |   -.020083   .1280266    -0.16   0.876    -.2743922    .2342262
        day2 |  -.1004775   .1293114    -0.78   0.439    -.3573387    .1563837
        day3 |  -.0038505   .1252044    -0.03   0.976    -.2525537    .2448527
        day4 |   .1026251   .1242444     0.83   0.411    -.1441711    .3494214
      speed3 |   .0201874   .0064022     3.15   0.002     .0074701    .0329046
       _cons |  -.6651257   .1516013    -4.39   0.000     -.966263   -.3639884
------------------------------------------------------------------------------

. predict phat1, xb

.
. reg lnqty day1 day2 day3 day4 mixed3 stormy3

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  6,    90) =    4.31
       Model |  12.5323209     6  2.08872015           Prob > F      =  0.0007
    Residual |  43.6001126    90  .484445695           R-squared     =  0.2233
-------------+------------------------------           Adj R-squared =  0.1715
       Total |  56.1324335    96  .584712849           Root MSE      =  .69602

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        day1 |  -.3171298    .229444    -1.38   0.170    -.7729603    .1387007
        day2 |  -.6655481   .2230281    -2.98   0.004    -1.108632   -.2224639
        day3 |  -.5577861   .2206741    -2.53   0.013    -.9961936   -.1193785
        day4 |  -.0345707    .221351    -0.16   0.876    -.4743231    .4051817
      mixed3 |  -.2615067   .1778435    -1.47   0.145    -.6148239    .0918104
     stormy3 |  -.5222825   .1686905    -3.10   0.003    -.8574155   -.1871494
       _cons |   8.650079   .1794426    48.21   0.000     8.293585    9.006573
------------------------------------------------------------------------------

. reg lnprice day1 day2 day3 day4 mixed3 stormy3

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  6,    90) =    3.36
       Model |  2.87271871     6  .478786451           Prob > F      =  0.0050
    Residual |  12.8403915    90  .142671017           R-squared     =  0.1828
-------------+------------------------------           Adj R-squared =  0.1283
       Total |  15.7131102    96  .163678231           Root MSE      =  .37772

------------------------------------------------------------------------------
     lnprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        day1 |  -.0094215   .1245151    -0.08   0.940    -.2567924    .2379495
        day2 |  -.0177709   .1210333    -0.15   0.884    -.2582247    .2226828
        day3 |   .0368483   .1197558     0.31   0.759    -.2010675    .2747642
        day4 |   .1245099   .1201232     1.04   0.303    -.1141358    .3631555
      mixed3 |   .2812021   .0965125     2.91   0.005     .0894632    .4729409
     stormy3 |   .3871992   .0915453     4.23   0.000     .2053286    .5690699
       _cons |  -.4866297   .0973803    -5.00   0.000    -.6800926   -.2931668
```


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```
------------------------------------------------------------------------------

. predict phat2, xb

. 
. // 2nd-stage estimates -- speed as instrument
. reg lnqty phat1 day1 day2 day3 day4

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  5,    91) =    4.77
       Model |  11.6510004     5  2.33020007           Prob > F      =  0.0006
    Residual |  44.4814331    91  .488806957           R-squared     =  0.2076
-------------+------------------------------           Adj R-squared =  0.1640
       Total |  56.1324335    96  .584712849           Root MSE      =  .69915

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       phat1 |  -1.566815   .5644648    -2.78   0.007    -2.688055    -.445575
        day1 |  -.2984291    .227249    -1.31   0.192    -.7498314    .1529731
        day2 |  -.6897303   .2240115    -3.08   0.003    -1.134702   -.2447589
        day3 |  -.4863624   .2225836    -2.19   0.031    -.9284975   -.0442274
        day4 |   .1657637   .2274403     0.73   0.468    -.2860185    .6175459
       _cons |   7.957192   .2205117    36.09   0.000     7.519173    8.395212
------------------------------------------------------------------------------

. // 2nd-stage estimates -- mixed3 and stormy3 as instruments
. reg lnqty phat2 day1 day2 day3 day4

      Source |       SS       df       MS              Number of obs =      97
-------------+------------------------------           F(  5,    91) =    5.10
       Model |  12.2836698     5  2.45673397           Prob > F      =  0.0004
    Residual |  43.8487636    91  .481854546           R-squared     =  0.2188
-------------+------------------------------           Adj R-squared =  0.1759
       Total |  56.1324335    96  .584712849           Root MSE      =  .69416

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       phat2 |  -1.268173    .419728    -3.02   0.003    -2.101911   -.4344354
        day1 |  -.3020106   .2255832    -1.34   0.184    -.7501038    .1460826
        day2 |  -.6877415    .222399    -3.09   0.003     -1.12951   -.2459732
        day3 |   -.499983    .220345    -2.27   0.026    -.9376713   -.0622948
        day4 |   .1375271   .2230704     0.62   0.539    -.3055748    .5806289
       _cons |   8.039471   .1935591    41.53   0.000      7.65499    8.423952
------------------------------------------------------------------------------

. 
. // same estimates using ivreg
. version 10 // (older versions -- just use "ivreg" in place of "ivregress
2sls
> ")

. ivregress 2sls lnqty day1 day2 day3 day4 (lnprice=speed3)

Instrumental variables (2SLS) regression               Number of obs =      97
```


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```
                                                      Wald chi2(5)  =   18.56
                                                      Prob > chi2   =  0.0023
                                                      R-squared     =       .
                                                      Root MSE      =  .79221

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnprice |  -1.566815   .6395994    -2.45   0.014    -2.820407   -.3132232
        day1 |  -.2984291   .2574976    -1.16   0.246    -.8031152    .2062569
        day2 |  -.6897303   .2538292    -2.72   0.007    -1.187226   -.1922342
        day3 |  -.4863624   .2522112    -1.93   0.054    -.9806874    .0079625
        day4 |   .1657637   .2577143     0.64   0.520    -.3393472    .6708745
       _cons |   7.957192   .2498635    31.85   0.000     7.467469    8.446916
------------------------------------------------------------------------------
Instrumented:  lnprice
Instruments:   day1 day2 day3 day4 speed3

. ivregress 2sls lnqty day1 day2 day3 day4 (lnprice=mixed3 stormy3)

Instrumental variables (2SLS) regression              Number of obs =      97
                                                      Wald chi2(5)  =   22.67
                                                      Prob > chi2   =  0.0004
                                                      R-squared     =  0.0635
                                                      Root MSE      =  .73618

------------------------------------------------------------------------------
       lnqty |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     lnprice |  -1.268173   .4451392    -2.85   0.004     -2.14063   -.3957166
        day1 |  -.3020106   .2392405    -1.26   0.207    -.7709133    .1668921
        day2 |  -.6877415   .2358635    -2.92   0.004    -1.150025   -.2254575
        day3 |  -.4999831   .2336852    -2.14   0.032    -.9579976   -.0419686
        day4 |   .1375271   .2365755     0.58   0.561    -.3261524    .6012066
       _cons |   8.039471   .2052776    39.16   0.000     7.637134    8.441808
------------------------------------------------------------------------------
Instrumented:  lnprice
Instruments:   day1 day2 day3 day4 mixed3 stormy3

.
. log close
       log:  /bbkinghome/paul_s/32/stataForLectures/ln18/ln18.log
  log type:  text
 closed on:  28 Apr 2009, 11:47:05
------------------------------------------------------------------------------
```


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