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The Interpretation of Instrumental Variables Estimators in Simultaneous Equations Models with an Application to the Demand for Fish

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In markets where prices are determined by the intersection of supply and demand curves, standard identification results require the presence of instruments that shift one curve but not the other. These results are typically presented in the context of linear models with fixed coefficients and additive residuals. The first contribution of this paper is an investigation of the consequences of relaxing both the linearity and the additivity assumption for the interpretation of linear instrumental variables estimators. Without these assumptions, the standard linear instrumental variables estimator identifies a weighted average of the derivative of the behavioural relationship of interest. A second contribution is the formulation of critical identifying assumptions in terms of demand and supply at different prices and instruments, rather than in terms of functional-form specific residuals. Our approach to the simultaneous equations problem and the average-derivative interpretation of instrumental variables estimates is illustrated by estimating the demand for fresh whiting at the Fulton fish market. Strong and credible instruments for identification of this demand function are available in the form of weather conditions at sea.

1. INTRODUCTION

Economists' interests in the analysis of competitive markets led to the development of statistical models for simultaneous equations. The results of this research programme, starting with Tinbergen (1930) and Haavelmo (1943), are now part of all econometric textbooks (*e.g.* Amemiya (1985), Goldberger (1991), Davidson and MacKinnon (1993))—see Hendry and Morgan (1997) for a historical perspective. In markets where prices are determined by the intersection of supply and demand curves, neither function can be consistently estimated by comparing average quantities traded at different values of observed (equilibrium) prices. Identification requires the presence of separate instruments that shift either demand or supply but not both. These results are typically presented in the context of linear models with fixed coefficients and additive residuals. Attempting to relax the assumptions that have no firm grounding in economic theory (*i.e.* linearity, additivity), researchers have considered identification in more general models. For example, Roehrig (1988) and Newey, Powell and Vella (1999) show that full identification requires the presence of continuously valued instruments. Even in this literature, however, additivity of residuals has typically been maintained.[1]

The first contribution of this paper is an investigation of the consequences of relaxing linearity and additivity assumptions for the interpretation of Instrumental Variables (IV) estimators. We show that standard IV methods estimate a weighted average of the derivative of the behavioural relationship of interest. In addition, we show that different instruments lead to different weighted average derivatives. We discuss how the choice of instruments can be governed by beliefs concerning the linearity and homogeneity of behavioural relationships, and the extent to which data can be used to guide the choice of instruments.

A second contribution is the formulation of critical assumptions underlying identification of heterogenous, time-varying demand functions in terms of potential demand and supply at different values of prices and instruments. This contrasts with most of the literature on simultaneous equations models, which casts critical assumptions in terms of unobservable functional-form-specific residuals. Our approach is similar to the potential outcome approach to evaluation questions originating in the statistical literature on causality (Rubin (1974, 1978), Holland (1986)) and builds on our earlier work on instrumental variables (Imbens and Angrist (1994), Angrist and Imbens (1995), Angrist, Imbens and Rubin (1996), and Imbens and Rubin (1997*a*, *b*)).

The theoretical ideas in the paper are illustrated in a re-analysis of wholesale demand for fresh whiting at the Fulton fish market in New York City, using data collected by Graddy (1995). This application is useful for our purposes because strong and credible instruments for identification of the demand function are available in the form of weather conditions at sea. The quantity of fish brought to market is determined by many factors, but the weather is an important determinant of supply since strong winds and high waves make it difficult to catch fish. At the same time, it is unlikely that weather conditions at sea affect the demand for fish.

2. THE MODEL

2.1. *Demand and supply functions*

In each of T markets, indexed by $t=1,\dots,T$, we postulate the existence of a demand and supply function, $q_t^d(p,z)$ and $q_t^s(p,z)$. These functions describe the quantities that would be demanded and supplied in market t for all possible prices p and for all possible values of a generic instrument z. Initially we take z to be binary. With no essential loss of generality we could write demand and supply in market t as a common function of prices, instruments and stochastic, market-specific residuals ε_t^d and ε_t^s, such that

1. Exceptions are Manski (1995, 1997) who investigates bounds on average demand and supply functions without functional form and additivity assumptions, and Pearl (1994) who uses graphical methods akin to path analysis.

$q_t^d(p,z)=q^d(p,z,\varepsilon_t^d)$ and $q_t^s(p,z)=q^s(p,z,\varepsilon_t^s)$. In Section 4 we exploit this second representation to make connections to the literature on conventional linear and non-linear simultaneous equations models. This representation is not used initially becase until the residuals are defined explicitly in terms of observable quantities, it is difficult to judge the plausibility of assumptions concerning their properties.

Demand or supply functions for any market are not observed. Rather, we observe quantities traded, and realized values for prices and instruments. Market clearing is assumed to determine which prices and quantities would be observed for any value of the instrument. Let $p_t^e(z)$ be the equilibrium price as a function of the instrument, implicitly defined by the market clearing condition equating demand and supply

$$q_t^d(p_t^e(z),z)=q_t^s(p_t^e(z),z). \tag{1}$$

Also define

$$q_t^e(z)\equiv q_t^d(p_t^e(z),z)=q_t^s(p_t^e(z),z), \tag{2}$$

to be the equilibrium quantity as a function of the instrument. We assume that for all z the equilibrium price and quantities exist and are unique in each market.

We observe the quadruple (z_t, p_t^e, q_t^e, x_t), where z_t is the value of the instrument, $p_t^e\equiv p_t^e(z_t)$ is the equilibrium price at the realized value of the instrument, $q_t^e\equiv q_t^e(z_t)$ is the quantity traded, and x_t is a vector of covariates. The covariates describe characteristics of markets that affect both supply and demand, as well as additional instruments. The covariates are included for two reasons. First, and most importantly, the assumptions underlying identification of demand and supply functions may be more plausible after conditioning on covariates. Second, covariates can make inference more precise.

The demand and supply functions are assumed to satisfy regularity conditions such as continuous differentiability in prices, but are otherwise unrestricted. Specifically the demand and supply functions are allowed to change freely from one market to the next according to some stochastic process. We assume their expected values or population averages, at fixed prices and instruments and conditional on covariates, are well defined

$$q^d(p,z\,|\,x)\equiv E[q_t^d(p,z)\,|\,x_t=x],$$

and

$$q^s(p,z\,|\,x)\equiv E[q_t^s(p,z)\,|\,x_t=x].$$

We also assume that these population averages are consistently estimable by averaging the demand and supply function at specific values for the prices and instruments over all markets with identical values of the covariates. Similarly, population-average equilibrium price and quantity functions are defined as

$$p^e(z\,|\,x)\equiv E[p_t^e(z)\,|\,x_t=x],$$

and

$$q^e(z\,|\,x)\equiv E[q_t^e(z)\,|\,x_t=x].$$

In our application, $q_t^d(p,z)$ may be, for example, the demand function on Monday December 6th, defined for all prices and weather conditions. One of the covariates in the application is the day of the week. The average demand function, $q^d(p,z\,|\,x)$ may be the average demand function as a function of prices and weather conditions, averaged over all Mondays.

2.2. *Instrumental variables assumptions*

We begin by laying out the assumptions regarding the relationship between the instruments and the heterogeneous and time-varying demand and supply functions. The first assumption is:

Assumption 1. (Independence).

$$z_t \perp\!\!\!\perp \{(q_t^d(p,z), q_t^s(p,z))\}_{p,z} | x_t.$$

(Throughout the paper $A \perp\!\!\!\perp B$ denotes independence of A and B, and $A \perp\!\!\!\perp B | C$ denotes conditional independence of A and B given C.) Assumption 1 means that the demand function evaluated at p and z should be independent of the instrument z_t. In our application, this amounts to assuming that although buyers and sellers come to the market with a demand and supply function that may depend on the weather and prices, the quantity demanded over the range of possible prices and weather conditions is independent of the value of the weather conditions actually realized. In other words, the weather is as good as randomly assigned given covariates.

An implication of this assumption is that

$$z_t \perp\!\!\!\perp \{p_t^e(z), q_t^e(z)\} | x_t,$$

for all z. Because the potential values of the equilibrium price $p_t^e(z)$ are independent of the realized value of the instrument z_t, the reduced form regression, that is, the average value of the equilibrium price as a function of instrument and covariates, can be estimated by averaging over markets with $z_t = z$ and $x_t = x$:

$$p^e(z|x) = E[p_t^e(z)|x_t = x] = E[p_t^e(z)|z_t = z, x_t = x] = E[p_t^e|z_t = z, x_t = x].$$

Here the first and third equality are by definition and the second by Assumption 1. A similar result holds for equilibrium quantities:

$$q^e(z|x) = E[q_t^e(z)|x_t = x] = E[q_t^e(z)|z_t = z, x_t = x] = E[q_t^e|z_t = z, x_t = x].$$

From this point on, we focus on the identification of the demand function. The argument for identification of the supply function is analogous. As in linear simultaneous equations models, a key identifying assumption is an exclusion restriction:

Assumption 2. (Exclusion).

$$q_t^d(p,z) = q_t^d(p,z'),$$

for all p, z, z', and t.

This assumption requires that, at any given price, there be no effect of the instrument on demand.[2] Given this assumption we can drop the instrument as argument in the demand function and define $q^d(p|x) \equiv q^d(p,z|x)$ for all z.

2. A weaker version of this assumption, similar to the exclusion restrictions used in Hirano, Imbens, Rubin and Zhou (1999), requires that

$$q_t^d(p,z_t) \perp\!\!\!\perp z_t | x_t, p_t^e(z),$$

for all p and z. This version of the exclusion restriction highlights the fact that in models with covariates only conditional distributions need to be identical. For clarity of exposition, however, we use the stronger version of the exclusion assumption.

The next assumption requires the instrument to impact upon prices.

Assumption 3. (Nonzero effect of instruments on prices).

$$\text{for some } x,\ p^e(z|x) \text{ is a non-trivial function of } z.$$

This guarantees that average prices in markets with instrument $z_t = 0$ and covariates $x_t = x$ differ from average prices in markets with instrument $z_t = 1$ and covariates $x_t = x$, at least for some values of x. Because the instrument does not shift demand by virtue of Assumption 2, any change in equilibrium prices must come from an effect of the instrument on the supply function.

The final assumption requires monotonicity of the equilibrium price in the instruments.

Assumption 4. (Monotonicity).

For all pairs (z, z') either

$$\Pr(p_t^e(z) \geqq p_t^e(z') | x_t) = 1,$$

or

$$\Pr(p_t^e(z) \leqq p_t^e(z') | x_t) = 1.$$

Without this condition the weights in our weighted average derivative representation of the instrumental variables estimand can be negative. In the standard linear model, monotonicity is a consequence of the constant-coefficients assumption. More generally, monotonicity requires that if for at least one market changing the instrument from $z = 0$ to $z = 1$ increases the equilibrium price, then this change does not decrease the equilibrium price for any other market.

Assumption 4 casts monotonicity in terms of the reduced form relationship between prices and instruments. A version of this assumption based on the underlying structural relationships is given in the following result.

Lemma 1. *If in all markets t,*

(i) *demand functions* $q_t^d(p)$ *are decreasing in prices, and do not depend on the instrument z,*
(ii) *supply functions* $q_t^s(p, z)$ *are increasing in prices, and*
(iii) *for all prices*

$$q_t^s(p, 1) \geqq (\leqq) q_t^s(p, 0),$$

then the monotonicity assumption is satisfied, and

$$p_t^e(1) \leqq (\geqq) p_t^e(0).$$

Proof. See Appendix. ||

Upward-sloping supply and downward-sloping demand functions are assumed in most economic discussions of demand and supply. Manski (1995, 1997) considers the implications of these assumptions in his analysis of bounds on demand and supply functions.

Although the assumptions of upward-sloping supply and downward-sloping demand are common, the third requirement in Lemma 1 is less often made explicit. In our application this assumption requires that on any given day, and for any price p, supply does not increase when the weather gets worse. Note that supply need not actually decrease in every market and at every price, as long as it decreases when the weather gets worse in some markets for some prices, as required by Assumption 3. However, we rule out a scenario with markets in which supply is high *because* the weather is bad.

3. MAIN RESULTS

3.1. *Identification of average derivatives of demand functions*

Next, we use Assumptions 1–4 to interpret differences in average quantities and prices at different values of the instruments. The following theorem presents the main result.

Theorem 1. *Suppose Assumptions* 1–4 *hold. Then the Instrumental Variables* (*IV*) *estimand, defined as the ratio of the difference in average quantities traded at two values of the supply instrument to the difference in average prices at the same two values of the supply instrument,*

$$\beta^*(x) = \frac{q^e(1\,|\,x) - q^e(0\,|\,x)}{p^e(1\,|\,x) - p^e(0\,|\,x)},$$

equals the following weighted average of the derivative of the demand function:

$$\beta^*(x) = \int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p)\,\middle|\, p_t^e(1) \geqq p \geqq p_t^e(0),\, x_t = x\right] \cdot \omega(p\,|\,x)dp,$$

where the weights

$$\omega(p\,|\,x) = \frac{\Pr(p_t^e(0) < p < p_t^e(1)\,|\,x_t = x)}{\int_0^\infty \Pr(p_t^e(0) < r < p_t^e(1)\,|\,x_t = x)dr},$$

are nonnegative and integrate to one.

Proof. See Appendix. ||

The result is stated for a given value of the covariates. In practice, it may not be feasible to estimate separate slopes for all values of the covariates and one may wish to average over the covariates. We return to this point in Section 3.5.

Theorem 1 describes the two types of averaging that characterize the instrumental variables estimand in simultaneous equations models. First, there is averaging over some of the markets at a given price. For any price p, only those markets whose equilibrium prices $p_t^e(0)$ and $p_t^e(1)$ bracket this price p enter into the expectation. The implication is that the more powerful an instrument is, the more markets will have equilibrium prices bracketing any given price (as reflected in the larger conditioning set $\{t\,|\,p_t^e(0) < p < p_t^e(1)\}$) and the more representative the IV estimand will be of the average demand function at p. Second, there is averaging over different prices in the same market. Averaging over different prices is reflected in the weighting function $\omega(\cdot)$. The weight given to any particular price is proportional to the number of markets whose equilibrium prices bracket this price.

Because of the monotonicity condition, the weight function can be written as

$$\omega(p\mid x)=\frac{\Pr(p<p_t^e(1)\mid x_t=x)-\Pr(p<p_t^e(0)\mid x_t=x)}{\int_0^\infty[\Pr(r<p_t^e(1)\mid x_t=x)-\Pr(r<p_t^e(0)\mid x_t=x)]dr}.$$

Note that

$$\Pr(p<p_t^e(1)\mid x_t=x)-\Pr(p<p_t^e(0)\mid x_t=x)$$
$$=\Pr(p<p_t^e\mid x_t=x, z_t=1)-\Pr(p<p_t^e\mid x_t=x, z_t=0),$$

and hence the numerator of the weight function can be estimated as the difference in the empirical distribution functions of p_t^e evaluated at p for all markets with $z_t=1$ and $z_t=0$.

An important aspect of Theorem 1 is that it does not require additive residuals, and is therefore not tied to a specific functional form. To illustrate this point, consider the corresponding result in logarithms. Let $\tilde{q}^e(z\mid x)=E[\ln q_t^e(z)\mid x_t=x]$, and $\tilde{p}^e(z\mid x)=E[\ln p_t^e(z)\mid x_t=x]$. Then we have.

Corollary 1. *Suppose Assumptions* 1–4 *hold. The Instrumental Variables* (*IV*) *estimand in logarithms,*

$$\tilde{\beta}(x)=\frac{\tilde{q}^e(1\mid x)-\tilde{q}^e(0\mid z)}{\tilde{p}^e(1\mid x)-\tilde{p}^e(0\mid x)},$$

equals:

$$\tilde{\beta}(x)=\int_0^\infty E\left[\frac{p}{q_t^d(p)}\frac{\partial q_t^d}{\partial p}(p)\,\middle|\,p_t^e(1)\geqq p\geqq p_t^e(0), x_t=x\right]\cdot\tilde{\omega}^d(p\mid x)dp,$$

a weighted average of the demand elasticity, where

$$\tilde{\omega}^d(p\mid x)=\frac{1/p\cdot\Pr(p_t^e(1)\geqq p\geqq p_t^e(0)\mid x_t=x)}{\int_0^\infty 1/r\cdot\Pr(p_t^e(1)\geqq r\geqq p_t^e(0)\mid x_t=x)dr}.$$

Proof. See Appendix. ||

Of course, if the demand elasticity is constant across price levels, so the logarithm of the market-specific demand function, $\ln q_t^d(p, z_t)$, is linear in log prices with constant coefficients, then the IV estimand is equal to this constant elasticity.

The results in Theorem 1 and Corollary 1 provide a clear causal interpretation for the standard linear instrumental variables estimator using a binary instrument. Another important issue is whether it is possible to identify other aspects of the average demand function, or its derivative, under Assumptions 1–4. Let the demand function in market t be $\bar{q}_t^d(p)$, with equilibrium price function $\bar{p}_t^e(z)$. Note that the derivative of the demand function in market t appears in the weighted average, β^*, only for values of p such that $\bar{p}_t^e(0)\leqq p\leqq\bar{p}_t^e(1)$. Now, suppose we change the demand functions from $\bar{q}_t^d(p,z)$ to $\tilde{q}_t^d(p,z)$ such that

$$\tilde{q}_t^d(p,z)\begin{cases}=\bar{q}_t^d(p,z) & \bar{p}_t^e(0)\leqq p\leqq\bar{p}_t^e(1),\\ \neq\bar{q}_t^d(p,z) & \bar{p}_t^e(0)>p \text{ or } p>\bar{p}_t^e(1),\end{cases}$$

without changing the supply functions. The old equilibrium price functions $\bar{p}_t^e(z)$ still clear markets, and therefore if equilibrium prices are unique, the new equilibrium price functions $\tilde{p}_t^e(z)$ are identical to the old equilibrium price functions: $\tilde{p}_t^e(z) = \bar{p}_t^e(z)$ for all t and p, and hence equilibrium quantity functions are also unchanged $\tilde{q}_t^e(z) = \bar{q}_t^e(z)$, for all t and p.

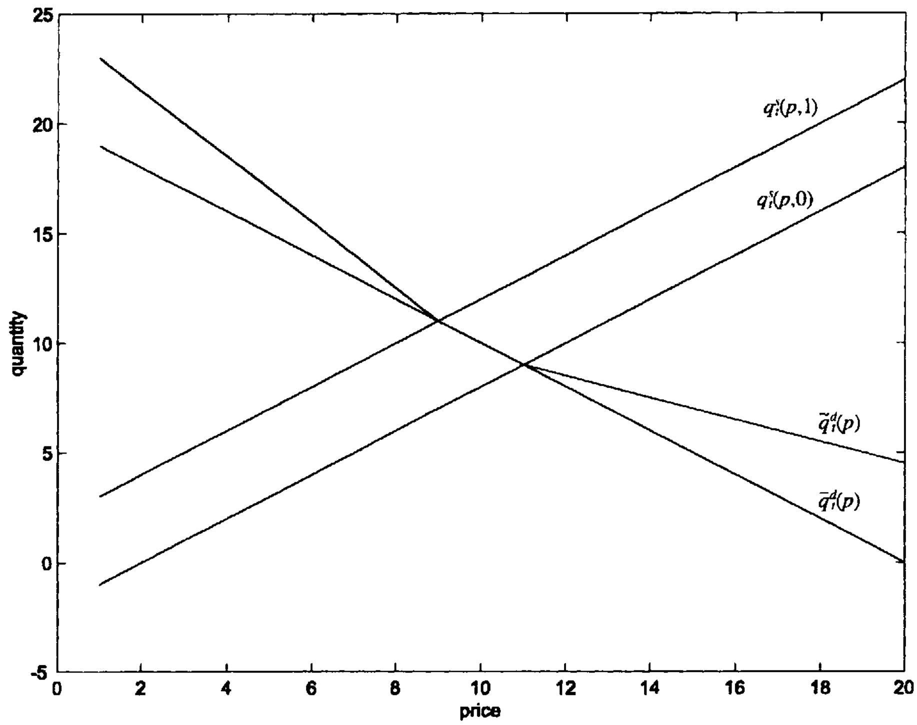

FIGURE 1

Changing demand functions outside the range of equilibrium prices

Figure 1 illustrates this point in a simple example with deterministic demand and supply functions where the only stochastic component comes from the instrument. The supply function is $q_t^s(p, z) = p + 4 \cdot z$. The first demand function is $\bar{q}_t^d(p) = 20 - p$. In periods where $z_t = 0$, the equilibrium price is 8 and the equilibrium quantity is 12. In period where $z_t = 1$, the equilibrium price is 10 and the equilibrium quantity is 10. The alternative demand function is

$$\tilde{q}_t^d(p) = \begin{cases} 24 - \frac{3}{2}p & p \leqq 8, \\ 20 - p & 8 < p < 10, \\ 15 - \frac{1}{2}p & p \geqq 11. \end{cases}$$

Between the equilibrium prices the two demand functions are identical, and therefore the equilibrium prices and quantities for any value of the instrument are identical, so there are no observable implications of the differences between the two demand functions.

In general, changing the demand function outside the range of equilibrium prices leaves equilibrium prices unchanged. Hence the joint distribution of instruments, equilibrium prices and quantities is not affected by the change from $\bar{q}_t^d(p)$ to $\tilde{q}_t^d(p)$. The new distribution of demand functions is therefore observationally equivalent to the old distribution, and differences between them are not identified. Therefore, we cannot identify average derivatives of the demand function that depend on derivatives at prices not included in the conditioning set in Theorem 1, that is, prices such that $p < p_t^e(0)$ or $p > p_t^e(1)$.

3.2. *Illustrating the two types of averaging*

This section uses two examples to illustrate the role played by nonlinearity and nonadditivity in determining the nature of the IV estimand under Theorem 1. In particular, we discuss how $\beta^*(x)$ is related to the population average demand function and its derivative. For ease of exposition, we assume there are no covariates.

First, suppose that demand and supply are linear in prices and instruments but with time-varying coefficients. As noted by Stoker (1993), such a model could arise as the consequence of aggregating linear demand functions that differ across groups in the population.[3] Changes in the average demand slope over time would then be attributable to changes in the distribution of the characteristics of buyers coming to market.

Corollary 2. *Suppose that Assumptions* 1–4 *hold, and that*

$$q_t^d(p) = \beta_{t0} + \beta_{t1} \cdot p,$$

$$q_t^s(p, z) = \alpha_{t0} + \alpha_{t1} \cdot p + \alpha_{t2} \cdot z.$$

Then

$$\beta^* = \frac{E[\beta_{t1} \cdot (p_t^e(1) - p_t^e(0))]}{E[p_t^e(1) - p_t^e(0)]} = E[\beta_{t1}] + \frac{\mathrm{Cov}\,(\beta_{t1}, -\alpha_{t2}/(\alpha_{t1} - \beta_{t1}))}{E[-\alpha_{t2}/(\alpha_{t1} - \beta_{t1})]}.$$

Proof. See Appendix. ||

In general, the IV estimand does *not* equal the slope of the average demand curve, $E[\beta_{t1}]$. Periods when $p_t^e(1) - p_t^e(0)$ is large get more weight in β^* than in $E[\beta_{t1}]$.

If we assume that the supply function has constant coefficients, that is, $\alpha_{t1} = \alpha_1$, and $\alpha_{t2} = \alpha_2 > 0$, then the covariance $\mathrm{Cov}\,(\beta_{t1}, -\alpha_{t2}/(\alpha_{t1} - \beta_{t1}))$ is positive, and the IV estimand β^* is closer to zero than $E[\beta_{t1}]$. The intuition for this result is that when β_{t1} is high in absolute value (demand is elastic), the price gap $p_t^e(1) - p_t^e(0) = -\alpha_2/(\alpha_1 - \beta_{t1})$, which weights each derivative, is small.

Now, suppose that demand is given by a non-linear relationship, with the only source of time-variation coming from an additive error term (as in, Roehrig (1988), Newey, Powell, and Vella (1999)).

Corollary 3. *Suppose Assumptions* 1–4 *hold and the demand function can be written as*

$$q_t^d(p) = q^d(p) + \varepsilon_t^d.$$

Then

$$\beta^* = \int_0^{\infty} \frac{\partial q^d}{\partial p}(p) \cdot \omega(p) dp,$$

3. For example, the results in Graddy (1996) suggest that Asian and white buyers at the Fulton fish market have different demand elasticities.

with weights

$$\omega(p) = \frac{\Pr(p_t^e(0) < p < p_t^e(1))}{\int_0^\infty \Pr(p_t^e(0) < r < p_t^e(1))dr}.$$

Proofs. See Appendix. ||

Thus, when the demand function can be written as invariant across periods except for an additive residual, β^* is a weighted average of the slope of the average demand function, $\partial q^d(p)/\partial p$. In this case, the slope of the demand function at a specific price involves no averaging because $\partial q_t^d(p)/\partial p = \partial q_s^d(p)/\partial p$ at all prices p and for all markets t and s. The weights reflect how likely the instruments are to shift the equilibrium price from below p to at least p.

3.3. *Choosing between two binary instruments*

Both types of averaging in Theorem 1 are specific to the instrument,. Two binary instruments can generate different β^*s, either because the demand function is nonlinear, or because there is heterogeneity in the slopes of the market-specific demand functions. When studying a specific policy question, one instrument may therefore be more valuable than another, even if both are valid in the sense of satisfying Assumptions 1–4. We illustrate this point here using a stylized example, focusing on estimation of a demand function with no covariates, using a single instrument that takes on three values. In Section 6 we apply these ideas to the Fulton fish market data.

Wholesale fish markets, like agricultural markets, are sometimes regulated, in which case a researcher may be interested in the difference in average demand at different prices. Consider comparing $p = 3$ versus $p = 2$, or $\beta^* \equiv q^d(3) - q^d(2)$. As in our empirical example, the researcher has a single instrument z_t (say, weather conditions) that takes on three values, $z_t \in \{\mathrm{f}, \mathrm{m}, \mathrm{s}\}$ (fair, mixed, and stormy). Finally, suppose that in all markets the fair weather equilibrium price is $p_t^e(\mathrm{f}) = 1$, the mixed weather equilibrium price is $p_t^e(\mathrm{m}) = 2$, in half the stormy weather equilibrium price is $p_t^e(\mathrm{s}) = 2$ and in the other half the stormy weather equilibrium price is $p_t^e(\mathrm{s}) = 3$. The average equilibrium prices are therefore $p^e(\mathrm{f}) = 1$, $p^e(\mathrm{m}) = 2$, and $p^e(\mathrm{s}) = 2 \times 1/2 + 3 \times 1/2 = 5/2$.

The researcher has a number of options in this case. First, she could use only the observations with fair or mixed weather. Based on Theorem 1, the binary instrument making a fair/mixed contrast estimates

$$\beta^{\mathrm{f/m}} = \frac{q^e(\mathrm{m}) - q^e(\mathrm{f})}{p^e(\mathrm{m}) - p^e(\mathrm{f})} = E[q_t^d(2) - q_t^d(1)] = q^d(2) - q^d(1).$$

Alternatively, the researcher could decide to use only the mixed/stormy weather data. The resulting instrumental variables estimand is

$$\beta^{\mathrm{m/s}} = \frac{q^e(\mathrm{s}) - q^e(\mathrm{m})}{p^e(\mathrm{s}) - p^e(\mathrm{m})} = E[q_t^d(3) - q_t^d(2) \mid p_t^e(\mathrm{s}) = 3].$$

The unnormalized weight functions for these two strategies are shown in Figure 2 (solid line for the weight function or $\beta^{\mathrm{f/m}}$ and dashed line for $\beta^{\mathrm{m/s}}$). For $\beta^{\mathrm{f/m}}$ the unnormalized weight function is

$$\Pr(p_t^e(\mathrm{f}) \leqq p) - \Pr(p_t^e(\mathrm{m}) \leqq p) = 1 \quad \text{for } 1 \leqq p \leqq 2,$$

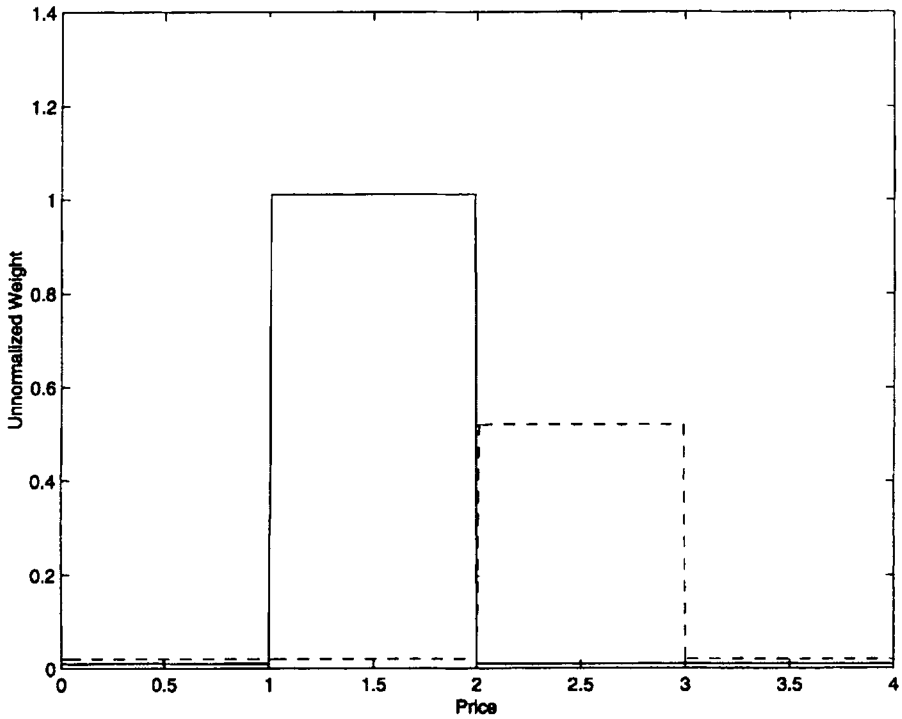

FIGURE 2
Weight function (artificial example)

and 0 otherwise, while for $\beta^{m/s}$ it is equal to

$$\Pr(p_t^e(m) \leqq p) - \Pr(p_t^e(s) \leqq p) = 1/2 \quad \text{for } 2 \leqq p \leqq 3,$$

and 0 otherwise. These differences in distribution functions can be estimated by the differences in the corresponding empirical distribution functions for the subpopulations with the instrument equal to fair, mixed and stormy respectively.

If the demand function is linear in each market, that is, if $q_t^d(p) = \beta_{t0} + \beta_{t1} \cdot p$ for all markets t, with possibly market-specific coefficients β_{0t} and β_{1t}, it follows that

$$\beta^{f/m} = E[q_t^d(2) - q_t^d(1)] = E[\beta_{t1}] = E[q_t^d(3) - q_t^d(2)] = \beta^*,$$

and the first strategy identifies the price effect of interest β^*, though $\beta^{m/s}$ would not necessarily do so. If, on the other hand, the demand function is identical in each market up to an additive shift ε_t^d, or $q_t^d(p) = q^d(p) + \varepsilon_t^d$, then

$$\begin{aligned}\beta^{m/s} &= E[q_t^d(3) - q_t^d(2) \mid p_t^e(s) = 3] \\ &= E[q^d(3) + \varepsilon_t^d - q^d(2) - \varepsilon_t^d] = q^d(3) - q^d(2) = \beta^*,\end{aligned}$$

and the second strategy identifies the price effect of interest. If demand functions are characterized by additive shifts as well as linearity in prices then both approaches identify the same price effect, β^*.

If demand functions are neither linear nor additive, the researcher must choose between $\beta^{f/m}$, which is a population average derivative but at the wrong prices, and $\beta^{m/s}$, which is the average derivative at the right prices but averaged only over a subset of the population of interest. Alternatively some combination of these estimates might be used. Two pieces of information are available to inform this decision. First, the difference between estimates based on different valid instruments is informative about the amount of nonlinearity and heterogeneity in demand functions. It would therefore be useful to know that the alternative IV estimates are close. Second, as in the above example, the weight functions are informative about the range that each IV estimator covers.

One implication of this discussion concerns the interpretation of over-identification tests for the validity of instruments. Such tests typically rely on comparisons of estimates based on different sets of instruments (*e.g.* Hausmann (1978)). In the current setting these tests could be performed by comparing $\beta^{f/m}$ and $\beta^{m/s}$. Traditionally, significant differences have been interpreted as evidence of violations of the instrumental variables assumptions, that is, of the independence or exclusion assumptions. This discussion suggests that differences between $\beta^{f/m}$ and $\beta^{m/s}$ can also arise from non-linearities and heterogeneity in demand function, even when all instruments are valid.

3.4. *Multiple instruments and two-stage-least-squares*

The example above involved a comparison between estimands each based on a single binary instrument. Often researchers use multi-valued or multiple instruments, combined in a two-stage-least-squares procedure. In the first stage of two-stage-least-squares estimation a single instrument is constructed based on the projection of the endogenous regressor (price in our application) on a set of instruments. In the second stage, the instrumental variables estimator is obtained as the ratio of the covariance of the dependent variable and the constructed instrument to the covariance of the endogenous regressor and the single constructed instrument. In this section we explore the interpretation of such procedures in the framework developed in this paper. Multiple, or multivalued, instruments add an additional layer of averaging. In addition to averaging over markets and prices, we now average over instruments. For ease of exposition covariates are ignored in this section.

We start by considering the case with discrete instruments. In that case the number of instruments is immaterial—only the number of distinct values of the vector of instruments matters. Let $\{z^0, \dots, z^K\}$ denote the set of possible values for the instruments. We assume that Assumptions 1–4 hold for all pairs (z^k, z^l). Without loss of generality, let the points of support be ordered by the value of the expected equilibrium price, that is $p^e(z^k) \leq p^e(z^{k+1})$ for all k. Define the IV estimand for each pair of instrument values

$$\beta^{z^k, z^l} = \frac{q^e(z^k) - q^e(z^l)}{p^e(z^k) - p^e(z^l)}.$$

This instrumental variables estimand can still be interpreted using Theorem 1.

Corollary 4. *Suppose Assumptions* 1–4 *hold for the pair of instrument values* (z^k, z^l) *with* $p_t^e(z^k) \leq p_t^e(z^l)$. *Then*

$$\beta^{z^k, z^l} = \int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p, z_t) \middle| p_t^e(z^k) \leq p \leq p_t^e(z^l)\right] \cdot \omega(p) dp, \tag{3}$$

with non-negative weights

$$\omega(p)=\frac{\Pr(p_t^e(z^l)\leqq p\leqq p_t^e(z^k))}{\int_0^\infty \Pr(p_t^e(z^l)\leqq r\leqq p_t^e(z^k))dr},$$

that integrate to one.

Proof. See Appendix. ||

Now consider the instrumental variables estimator based on the ratio of the covariance of equilibrium quantities and some function $g(\cdot)$ of the instruments, to the covariance of equilibrium prices and the same function $g(\cdot)$ of the instruments:

$$\beta^g\equiv\frac{\operatorname{Cov}(q_t^e, g(z_t))}{\operatorname{Cov}(p_t^e, g(z_t))}. \tag{4}$$

The function $g(\cdot)$ may be based on a first stage regression of prices on a set of instruments, or any other function of the instruments. Estimation of $g(\cdot)$ does not affect the probability limit of the estimator, although it may affect its distribution. The following theorem gives an interpretation for β^g in nonlinear, heterogenous models.

Theorem 2. *Suppose Assumptions* 1–4 *hold for all pairs of instruments* (z^k, z^l). *Then*

$$\beta^g=\textstyle\sum_{k=1}^K \lambda_k\cdot\beta^{z^{k-1},z^k},$$

with

$$\lambda_k=\frac{(p^e(z^k)-p^e(z^{k-1}))\cdot\sum_{l=k}^K f_z(z^l)\cdot(g(z^l)-E[g(z_t)])}{\sum_{m=1}^K(p^e(z^m)-p^e(z^{m-1}))\cdot\sum_{l=m}^K f_z(z^l)\cdot(g(z^l)-E[g(z_t)])},$$

where $f_z(z)$ *is the probability mass function of the instrument.*

Proof. See Appendix. ||

The weights in this case, λ_k, are non-negative as long as ordering the instruments by expected equilibrium price simultaneously orders the instruments by the value of $g(z)$. This happens, for example, if $g(z)=p^e(z)$. Note also that the weights are proportional to the first stage impact on prices at each successive contrast.

We have worked with discrete instruments so far. Now let demand, supply, equilibrium price and quantity functions and their expectations be continuously differentiable in a continuous, scalar, instrument z. In this case, monotonicity requires that the derivative of equilibrium price functions with respect to the instruments, at any value of the instrument, be either non-negative or non-positive for all markets. Define the limit of the binary instrumental variables estimands as follows

$$\beta(z)=\lim_{dz\downarrow 0}\beta^{z,z+dz}.$$

Theorem 3. *Suppose Assumptions* 1–4 *hold for all pairs of values of the instrument. Then*

$$\beta(z)=E\left[\frac{\partial q_t^d}{\partial p}(p_t^e(z))\right].$$

Proof. See Appendix. ||

Now consider the ratio of the covariance of equilibrium quantities and some function of a continuous instrument to the covariance of equilibrium prices and the same function of the instrument, as in equation (4). The following theorem provides an interpretation for this estimand.

Theorem 4. *Suppose Assumptions* 1–4 *hold for all pairs of values of the instrument. Then*

$$\beta^g = \frac{\operatorname{Cov}(q_t^e, g(z_t))}{\operatorname{Cov}(p_t^e, g(z_t))} = \int \beta(z) \cdot \lambda(z) dz,$$

with weight function

$$\lambda(z) = \frac{\dfrac{\partial p^e}{\partial z}(z) \cdot \displaystyle\int_z^\infty (g(y) - E[g(z_t)]) \cdot f_z(y) dy}{\displaystyle\int \frac{\partial p^e}{\partial z}(v) \cdot \int_v^\infty (g(y) - E[g(z_t)] \cdot f_z(y) dy dv}.$$

Proof. See Appendix. ||

Again, the weights $\lambda(z)$ in this result are non-negative if $g(z)$ is equal to the expected value of the price given the instrument. In the standard linear simultaneous equations model the choice of $g(\cdot)$ is usually determined by efficiency considerations, since different choices of $g(\cdot)$ in that context all estimate the same quantity. Here different choices of $g(\cdot)$ correspond to different estimands.

3.5. *Estimation with covariates*

One simple approach to controlling for covariates is to carry out the analysis separately for all values of the covariates and then report averages using the marginal or some conditional distribution of the covariates. With a small sample and detailed covariates such an approach is unlikely to be satisfactory. We therefore incorporate covariates by assuming that they enter the average equilibrium price and quantity functions additively and linearly.

Assumption 5. The average equilibrium price and quantity are linear and additive in covariates.

$$p^e(z|x) = p_0^e(z) + \gamma_p \cdot x,$$

$$q^e(z|x) = q_0^e(z) + \gamma_q \cdot x.$$

This assumption can be used to justify standard estimation procedures where in a first stage the endogenous regressor (price in this case) is regressed on the instrument and covariates, and in the second stage the outcome (quantity in this case) is regressed on the predicted price and covariates.

Lemma 2. *Suppose Assumptions* 1–5 *hold. Then the ratio of the expected value of the instrument coefficient from a linear regression of the quantity on the instrument and* *covariates to the expected value of the instrument coefficient from a linear regression of the price on the instrument and covariates is equal to*

$$\beta^* = \int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p)\middle| p_t^e(z') \geqq p \geqq p_t^e(z), x_t = x\right] \cdot \omega^d(p|x)dp,$$

and does not depend on x.

Proof. See Appendix. ||

The most natural way to motivate the additive linear structure in Assumption 5 is through linearity and additivity assumptions on the market specific demand and supply functions. In the application, we apply Assumption 5 to a model in logarithms.

4. ADDITIVITY AND LINEARITY IN TRADITIONAL SIMULTANEOUS EQUATION MODELS

4.1. *Additive residuals*

This section explores the relationship between the approach in Sections 2 and 3 and standard simultaneous equations models (SEM), which present critical assumptions in terms of observed variables and unobserved residuals. Our starting point is a set of unrestricted demand and supply functions $q_t^d(p, z)$ and $q_t^s(p, z)$, allowed to vary freely across markets. Alternatively, we could write $q^d(p, z; \varepsilon_t^d)$ and $q^s(p, z; \varepsilon_t^s)$ and formulate assumptions in terms of the properties of the residuals ε_t^d and ε_t^s as in the traditional SEM. This raises the question of what exactly these residuals represent in our framework. Most discussions of SEM assume an additive model for residuals.

Assumption 6. (Additive residuals).

$$q_t^d(p, z) = q^d(p, z|x_t) + \varepsilon_t^d,$$

$$q_t^s(p, z) = q^s(p, z|x_t) + \varepsilon_t^s,$$

at all prices p, instruments z and for all markets t, where ε_t^d and ε_t^s are defined as

$$\varepsilon_t^d \equiv q_t^e - q^d(p_t^e, z_t|x_t), \qquad \varepsilon_t^s \equiv q_t^e - q^s(p_t^e, z_t|x_t).$$

By definition, the demand residual satisfies

$$q_t^d(p, z) = q^d(p, z|x_t) + \varepsilon_t^d,$$

at $(p, z) = (p_t^e, z_t)$. The additivity assumption requires this equation to hold at all other values of the instrument and prices as well. Additivity therefore restricts the cross-market heterogeneity in demand and supply functions, implying that the slope of the average and the market-specific demand functions are identical.

Given additive residuals, our independence assumption (Assumption 1) is equivalent to the following textbook version:

Assumption 7. (Independence).

$$(\varepsilon_t^d, \varepsilon_t^s) \;\#\; z_t | x_t.$$

Without additive residuals, however, the residuals defined as $\varepsilon_t^d \equiv q_t^e - q^d(p_t^e, z_t | x_t)$ and $\varepsilon_t^s \equiv q_t^e - q^s(p_t^e, z_t | x_t)$ do not necessarily have this property. Consider the following example:

$$q_t^s(p, z) = \alpha_0 + \alpha_1 \cdot p + \alpha_2 \cdot z,$$
$$q_t^d(p, z) = \beta_{0t} + \beta_{1t} \cdot p.$$

Let β_{0t} and β_{1t} be independent of z_t with expectations $\bar{\beta}_0$ and $\bar{\beta}_1$. Using the definition above the two residuals are

$$\varepsilon_t^s = 0,$$

and

$$\varepsilon_t^d = \beta_{0t} - \bar{\beta}_0 + (\beta_{1t} - \bar{\beta}_1) \cdot p_t^e = \beta_{0t} - \bar{\beta}_0 + (\beta_{1t} - \bar{\beta}_1) \cdot \left(\frac{\alpha_0 - \beta_0}{\beta_{1t} - \alpha_1} + z_t \cdot \frac{\alpha_2}{\beta_{1t} - \alpha_1} \right).$$

Unless $\beta_{1t} - \bar{\beta}_1 = 0$, this residual is *not* independent of z_t.

The assumption of additive residuals is difficult to assess because it is closely tied to functional form. Assumptions formulated in terms of residuals as defined above typically do not remain valid after transforming the dependent variable. For example, if the model is additive in levels, it cannot be additive in logarithms and *vice versa*. Roehrig (1988) recognizes the importance of this issue, without providing any solutions: "For once one is willing to recognize the nonlinearities of the real world, one must begin to question the assumption of a simple additive error term that is distributed independently of the exogenous variables. This paper provides some definite answers under the assumption of such an error term but does not address the validity of this assumption." (p. 447)

The assumption of additive residuals can sometimes be tested in our framework. Consider a variation on the example in Section 3.3 where $z \in \{\mathrm{f}, \mathrm{m}, \mathrm{s}\}$, $p_t^e(\mathrm{f}) = 1$, $p_t^e(\mathrm{s}) = 2$, and $\Pr(p_t^e(\mathrm{m}) = 1) = \Pr(p_t^e(\mathrm{m}) = 2) = 1/2$. Then

$$\beta^{\mathrm{f/m}} = \frac{q^e(\mathrm{m}) - q^e(\mathrm{f})}{p^e(\mathrm{m}) - p^e(\mathrm{f})} = E[q_t^d(2) - q_t^d(1) | p_t^e(\mathrm{m}) = 1],$$

and

$$\beta^{\mathrm{m/s}} = \frac{q^e(\mathrm{s}) - q^e(\mathrm{m})}{p^e(\mathrm{s}) - p^e(\mathrm{m})} = E[q_t^d(2) - q_t^d(1) | p_t^e(\mathrm{m}) = 2].$$

Both instruments move half the markets from an equilibrium price equal to 1 to an equilibrium price equal to 2. However, the markets they affect are different. With the fair/mixed instrument the markets averaged in the estimand are those with $p_t^e(\mathrm{m}) = 1$, while with the mixed/stormy instrument the markets averaged are those with $p_t^e(\mathrm{m}) = 2$. With additive residuals, the conditioning set does not affect the estimand and the two estimands are identical. Testing the equality of the estimates based on the different instruments can, conditional on the validity of the instruments, be interpreted as a test of additivity.

4.2. *Linear simultaneous equations models*

A second component of the traditional framework is linearity.

Assumption 8. (Linearity in prices and instruments).

$$q_t^d(p, z) = \beta_{t0} + \beta_{t1} \cdot p + \beta_{t2} \cdot z,$$

$$q_t^s(p, z) = \alpha_{t0} + \alpha_{t1} \cdot p + \alpha_{t2} \cdot z.$$

A third commonly used assumption requires the effect of covariates to be additive and linear in the structural equations.

Assumption 9. (Linearity and additivity in covariates).

$$q^d(p, z | x) = q_0^d(p, z) + \beta_3 \cdot x,$$

$$q^s(p, z | x) = q_0^s(p, z) + \alpha_3 \cdot x,$$

An implication of linearity and additivity is summarized in the following lemma.

Lemma 3. *Suppose Assumptions* 6, 8, *and* 9 *hold. Then the demand and supply equations have the form*

$$q_t^d(p, z) = \beta_0 + \beta_1 \cdot p + \beta_2 \cdot z + \beta_3 \cdot x_t + \varepsilon_t^d, \tag{5}$$

$$q_t(p, z) = \alpha_0 + \alpha_1 \cdot p + \alpha_2 \cdot z + \alpha_3 \cdot x_t + \varepsilon_t^s. \tag{6}$$

Proof. See Appendix. ||

The nonlinear models considered by Roehrig (1988) and Newey, Powell and Vella (1994), relax the linearity assumptions (Assumptions 8 and 9) but maintain additivity of residuals (Assumption 6).

How does the textbook model fit our framework? Maintaining the LSEM assumptions 8–9, the earlier assumptions 1–4 can be reformulated in the textbook form as follows.

Assumption 10. (Exclusion). The coefficient on the instrument in the demand function, β_2, equals zero.

Assumption 11. (Nonzero effect of instruments on prices). The coefficient on the instrument in the supply function α_2, differs from zero.

Lemma 4. (*Equivalence*)

(i) *Given Assumption* 6, *Assumption* 7 *is equivalent to Assumption* 1.
(ii) *Given Assumptions* 6, 8 *and* 9, *Assumption* 10 *is equivalent to Assumption* 2.
(iii) *Given Assumptions* 6, 8, 9 *and* 10, *Assumption* 11 *is equivalent to Assumption* 3.
(iv) *Given Assumptions* 6, 8 *and* 9, *Assumption* 4 *is automatically satisfied.*

Proof. See Appendix. ||

Note that in textbook discussions, Assumptions 7 and 10 are often combined in a single assumption requiring that ε_t^d be independent of z_t in a demand function that already omits the instrument as an argument.

5. THE DEMAND FOR WHITING AT THE FULTON FISH MARKET

5.1. *The Fulton fish market*

Fish is sold by about 35 different dealers at the Fulton fish market, although only six of the dealers regularly sell whiting. There are no posted prices, and each dealer is free to charge a different price to each customer. Dealers can leave the Fulton market and new dealers can rent stalls, although in practice this happens rarely and did not happen over the sample period. The buyers at the Fulton market generally own retail fish shops or restaurants.

Whiting is a good choice for a study of the wholesale fish market because more transactions take place in whiting than almost any other fish. Whiting also vary less in size and quality than other fish. Finally, there is probably very little substitution between whiting and other fish. Whiting is a very cheap fish in large supply that is oily and distinctive tasting. Other fish would rarely be sold at a low enough price and in sufficient quantities to be attractive to retailers or restaurants as a substitute for whiting.

The data used in Graddy (1995) were obtained from a single dealer who supplied his inventory sheets for the period December 2nd, 1991 through May 8th, 1992. Total price and quantity for each transaction are recorded on the inventory sheets. These data are supplemented by data that were collected by direct observation from the same dealer during the period April 13th through May 8th, 1992. For this study, the prices and quantities are aggregated by day, for the 111 days the market was open between December 2nd and May 8th. The price variable used below is the quantity-weighted average daily transaction price for the dealer observed. The quantity variable is the total quantity sold by this dealer on each day. Table 1 presents summary statistics for the data used in this paper.

Every day the demand for fish at the Fulton fish market is determined partly by which customers decide to visit the market that day, as well as by how much they buy. A number of customers visit the market every week on Mondays and Thursdays, and other customers may visit the market every day of the week. Quantities purchased by individual

TABLE 1

Summary statistics (111 *Obs.*)

Variable	mean	s.d.	min	max
log (average daily price)	−0·19	0·38	−1·11	0·66
log (quantity)	8·52	0·74	6·19	9·98
Stormy	0·29	0·46	0	1
Mixed	0·31	0·46	0	1
Fair	0·41	0·49	0	1
Rainy on shore	0·16	0·37	0	1
Cold on shore	0·50	0·50	0	1
Monday	0·19	0·39	0	1
Tuesday	0·21	0·41	0	1
Wednesday	0·19	0·39	0	1
Thursday	0·21	0·41	0	1
log (minimum daily price)	−0·57	0·53	−2·30	0·41
log (maximum daily price)	0·06	0·31	−0·51	0·92
log (median daily price)	−0·20	0·41	−1·39	0·69
Daily price range	0·48	0·28	0·00	2·00

Notes: Prices and quantities are daily observations for whiting at the Fulton fish market. Prices are dollars per pound and quantities are pounds per day.

wholesale customers with potentially different demand functions are aggregated to produce the daily aggregate demand for fish. Aggregation is therefore one source of shifts in potential demand over time. For example, Graddy (1995) presents evidence which suggests that Asian buyers have a more elastic demand for fish than Whites and that the ethnic mix of buyers changes from day to day.

We begin our analysis with a demand function for the quantity demanded by customer c on day t

$$q^d_{tc}(p).$$

The subscript c ranges over the list of customers who ever visited the Fulton fish market during our sample period ($c = 1, \ldots, C$). The total demand for market t is

$$q^d_t(p) = \sum_{c=1}^{C} q^d_{tc}(p).$$

There are two complications in applying the methods discussed in this paper to the fish market data. First, not all buyers pay the same price per pound of whiting. We ignore this complication and assume that all buyers on the same day pay the same price p^e_t, constructed as the average of the individual transaction prices weighted by the quantities traded

$$p^e_t = \sum_{c=1}^{C} p^e_{tc} \cdot q^e_{tc} / \sum_{c=1}^{C} q^e_{tc}.$$

Second, we only have data for one of the dealers. We assume that this dealer has a constant share of the market, so we can analyse his quantities as if they were market quantities.

5.2. *Reduced-form estimates*

Tables 2 and 3 report reduced-form estimates of the relationship between log prices and quantities and weather conditions. These are estimates of the functions $p^e(z, x)$ and $q^e(z, x)$ for various choices of instruments and covariates. The dummy instrument, Stormy, indicates wave height greater than 4·5 feet and wind speed greater than 18 knots. A second dummy instrument, Mixed, indicates wave height greater than 3·8 feet, wind speed greater than 13 knots when Stormy equals zero. Wind speed and wave height are moving averages of the last three days' wind speed and wave height before the trading day, as measured off the coast of Long Island and reported in the New York Times boating forecast.

The reduced-form results show that Stormy is a statistically significant determinant of both the price and quantity of fish sold at the Fulton market. Stormy weather decreases

TABLE 2

Reduced form estimates for log quantity (111 *Obs.*)

Variable	coef	(s.e.)	coef	(s.e.)	coef	(s.e.)	coef	(s.e.)
Stormy	−0·36	(0·15)	−0·38	(0·15)	−0·45	(0·08)	−0·43	(0·17)
Mixed					−0·20	(0·17)	−0·10	(0·16)
Monday			0·12	(0·19)			0·11	(0·19)
Tuesday			−0·46	(0·19)			−0·45	(0·19)
Wednesday			−0·54	(0·20)			−0·53	(0·20)
Thursday			0·07	(0·16)			0·07	(0·16)
Good weather on shore			0·09	(0·15)			0·08	(0·17)
Rain on Shore			−0·03	(0·14)			−0·02	(0·14)

Note: All covariates and instruments are dummy variables.

TABLE 3

Reduced form estimates for log price (111 *Obs*)

Variable	coef	(s.e.)	coef	(s.e.)	coef	(s.e.)	coef	(s.e.)
Stormy	0·34	(0·07)	0·31	(0·08)	0·44	(0·08)	0·42	(0·08)
Mixed					0·24	(0·08)	0·23	(0·08)
Monday			−0·12	(0·11)			−0·11	(0·11)
Tuesday			−0·06	(0·11)			−0·08	(0·11)
Wednesday			−0·03	(0·11)			−0·06	(0·11)
Thursday			0·04	(0·11)			0·03	(0·11)
Good weather on shore			−0·02	(0·10)			−0·01	(0·09)
Rain on shore			0·08	(0·07)			0·06	(0·07)

Note: Standard errors are reported in parentheses.

the quantity and increases the price. The covariates are not significant in the price equation, but they do show that the quantity traded on Tuesday and Wednesday is significantly less than that traded on other days of the week. The coefficients on Mixed and Stormy are increasing in absolute value, and jointly significant.

5.3. *Estimates of the average elasticity*

The next table reports estimates of the weighted average demand elasticity. The first column reports estimates of

$$\tilde{\beta}^{\text{ns,s}} = \frac{\tilde{q}^e(\text{s}) - \tilde{q}^e(\text{ns})}{\tilde{p}^e(\text{s}) - \tilde{p}^e(\text{ns})}.$$

We also present estimates comparing fair and mixed, and mixed and stormy weather:

$$\tilde{\beta}^{\text{f,m}} = \frac{\tilde{q}^e(\text{m}) - \tilde{q}^e(\text{f})}{\tilde{p}^e(\text{m}) - \tilde{p}^e(\text{f})} \quad \text{and} \quad \tilde{\beta}^{\text{m,s}} = \frac{\tilde{q}^e(\text{s}) - \tilde{q}^e(\text{m})}{\tilde{p}^e(\text{s}) - \tilde{p}^e(\text{m})},$$

respectively. Note that these estimates also correspond to the instrumental variables estimates of β_1 in a standard setup with the demand function equal to

$$\ln q_t^d(p) = \beta_0 + \beta_1 \cdot \ln p + \varepsilon_t^d,$$

using only observations with $z_t = z$ and $z_t = z'$ and using the binary indicator for $z_t = z'$ as the instrument.

Table 4 shows estimates of $\tilde{\beta}(z, z', x)$ for the same combinations of z and z' as before. The covariates include indicators for days of the week and for weather on shore. These estimates are numerically identical to the instrumental variables estimates of β_1 in text –

TABLE 4

Instrumental variables estimates of demand function

	Stormy/Not-stormy				Fair/Mixed				Mixed/Stormy			
Sample size	111				79				66			
Av. price effect	−1·08	(0·48)	−1·22	(0·55)	−0·85	(0·70)	−0·31	(0·59)	−1·24	(0·97)	−1·26	(0·87)
Monday			−0·03	(0·17)			0·13	(0·21)			−0·00	(0·34)
Tuesday			−0·53	(0·18)			−0·54	(0·20)			−0·51	(0·30)
Wednesday			−0·58	(0·20)			−0·69	(0·18)			−0·54	(0·28)
Thursday			0·12	(0·18)			−0·03	(0·19)			0·26	(0·24)
Weather on shore			0·07	(0·16)			−0·02	(0·18)			−0·09	(0·23)
Rain on shore			0·07	(0·16)			0·23	(0·22)			−0·22	(0·17)

Notes: The excluded instruments are shown in the column headings. Standard errors are reported in parentheses.

TABLE 5

Two-stage-least-squares estimates of demand function with stormy and mixed as instruments

Variable	est.	(s.e.)	est.	(s.e.)
Av. price effect	−1·01	(0·42)	−0·947	(0·46)
Monday			−0·013	(0·18)
Tuesday			−0·51	(0·18)
Wednesday			−0·56	(0·17)
Thursday			0·10	(0·17)
Weather on shore			0·02	(0·16)
Rain on shore			0·07	(0·16)

Note: Standard errors are reported in parentheses.

book versions of instrumental variables with the demand function equal to

$$\ln q_t^d(p) = \beta_0 + \beta_1 \cdot \ln p + \beta_2 \cdot x + \varepsilon_t^d,$$

again using observations with $z_t = z$ and $z_t = z'$ and using the indicator for $z_t = z'$ as the instrument. Note that under the additivity assumption (Assumption 5) the value of $\tilde{\beta}(z, z', x)$ does not vary with x.

Table 5 presents two-stage-least-squares estimates using both the stormy and mixed instruments, with and without the additional regressors. These estimates are close to those based on the single instruments, with slightly lower standard errors.

Table 6 provides some evidence on the robustness of the results. First we replace average daily prices with the minimum, maximum, and the median of daily prices. Second, we split the sample by period and transaction volume. The estimates do not vary much across specifications.

5.4. *Choosing between different instruments*

This section discusses the choice of instruments. Figure 3 plots the three empirical log price distribution functions, estimates of $F_z(p) = \Pr(\log p_t^e(z) \leqq p)$, for $z \in \{\text{f, m, s}\}$. The difference between these distribution functions, presented in Figure 4, is equal to the unnormalized weight function for the estimands corresponding to using the fair/mixed data only (solid line), and using the mixed/stormy data only (dashed line). In addition the estimated demand functions for each of the two subsets of the data are presented in this figure, assuming linearity. The estimated slope of the demand function in these two cases is −0·85 and −1·24 respectively.

TABLE 6

Instrumental variables estimates of demand function using stormy/not stormy as instrument (alternative specifications)

	Min Price		Max Price		Median Price		First Half		Second Half		Low Vol.		High Vol.	
Sample size	111		111		111		56		55		55		56	
Price	−1·46	(0·87)	−1·29	(0·54)	−1·15	(0·52)	−1·87	(0·82)	−1·27	(0·79)	−1·00	(1·10)	−1·52	(0·55)
Mon	0·18	(0·21)	−0·04	(0·20)	−0·04	(0·18)	−0·14	(0·27)	−0·00	(0·24)	0·09	(0·36)	−0·15	(0·29)
Tue	−0·47	(0·26)	−0·50	(0·20)	−0·54	(0·19)	−0·63	(0·27)	−0·53	(0·32)	−0·28	(0·29)	−0·76	(0·24)
Wed	−0·43	(0·22)	−0·58	(0·17)	−0·56	(0·17)	−0·68	(0·37)	−0·52	(0·25)	−0·40	(0·33)	−0·65	(0·25)
Thur	0·27	(0·22)	0·07	(0·18)	0·13	(0·18)	−0·18	(0·24)	0·32	(0·24)	0·32	(0·47)	−0·11	(0·16)
Weath.	−0·06	(0·19)	0·08	(0·18)	0·06	(0·15)	0·13	(0·30)	−0·07	(0·16)	−0·04	(0·36)	0·16	(0·19)
Rain	0·30	(0·26)	0·03	(0·16)	0·09	(0·16)	−0·15	(0·27)	0·21	(0·20)	−0·11	(0·23)	−0·11	(0·34)

Notes: The column headings describe alternative choices of endogenous regressor and subsamples. Standard errors are reported in parentheses.

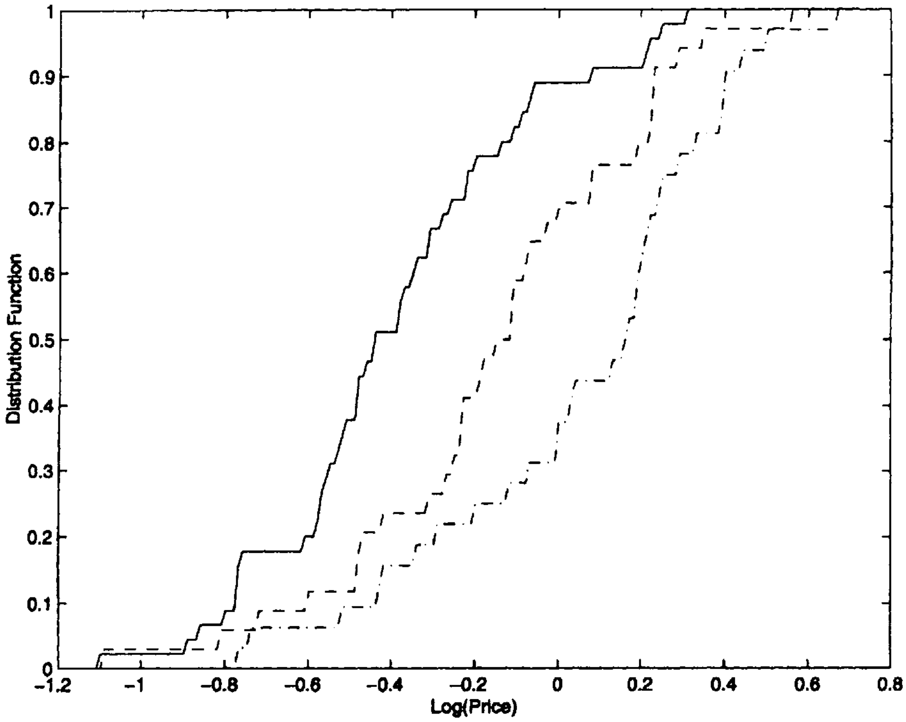

FIGURE 3
Empirical distribution function of log prices by weather conditions

The fair/mixed and mixed/stormy weight functions capture different price ranges. The fair/mixed instrument estimates a weighted average of the demand function with most weight attached to prices between 55 and 100 cents per pound (log prices between $-0{\cdot}6$ and $0{\cdot}0$). The mixed/stormy instrument estimates a weighted average of the demand function with most weight attached to prices between 82 and 150 cents (log prices between $-0{\cdot}2$ and $0{\cdot}4$). Neither instrument captures more than about a third of the total number of markets at any particular price. (Recall that the height of the unnormalized weight function at price p is an estimate of the proportion of the markets where equilibria span p). The estimates may be of little value for predicting effects in markets where weather conditions affect prices below 50 cents or above 150 cents, or in markets where weather does not affect prices. On the other hand, the fact that the alternative estimates are reasonably close suggests that heterogeneity in demand may be limited.

6. CONCLUSION

This paper investigates the consequences of relaxing additivity and linearity assumptions in simultaneous equations models. We show that without these assumptions, instrumental variables methods still estimate weighted average derivatives of the functions of interest. Averaging is across markets and along the nonlinear demand and supply function. The exact nature of the averaging depends on the relationship between the equilibrium prices and the instruments. The estimated weighted average therefore depends on the instrument employed. On the other hand, much can be learned about the range of price variation

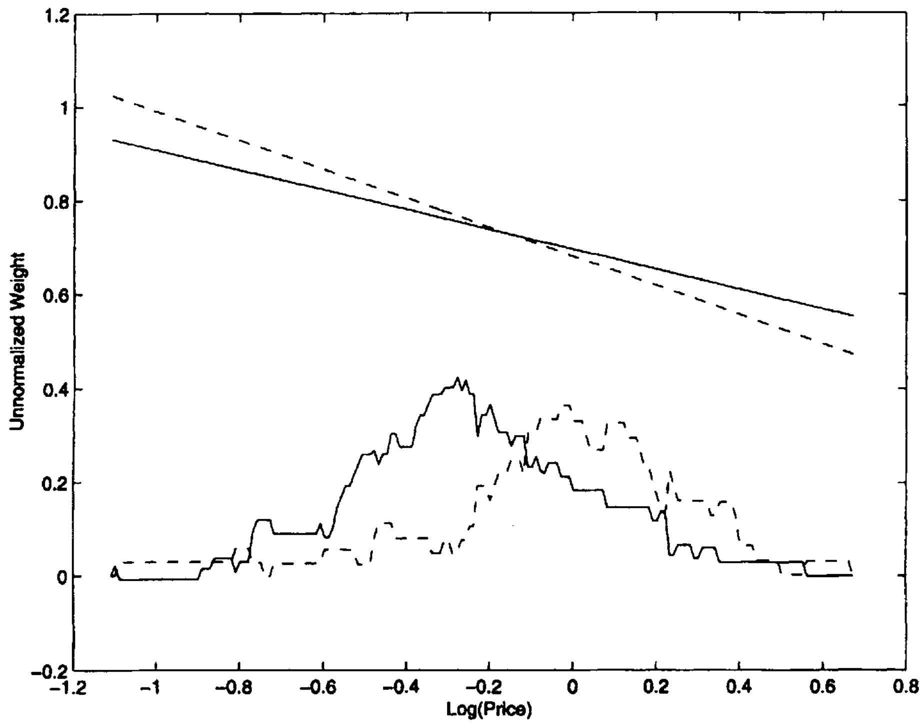

FIGURE 4

Weight and regression functions for different binary instruments

underlying each estimate, and the proportion of markets affected by the instrument at each price. These results are obtained by formulating the simultaneous equations model in terms of demand and supply at different prices and instruments, rather than in terms of functional-form specific residuals. We illustrate the assumptions and theoretical results by estimating the demand for Whiting at the Fulton fish market.

APPENDIX

Proof of Lemma 1.

Without loss of generality suppose $q^s_t(p,z) \geqq q^s_t(p,z')$ for all p. Define the excess supply function

$$e^s_t(p,z)=q^s_t(p,z)-q^d_t(p).$$

By definition of the equilibrium price $p^e_t(z)$,

$$e^s_t(p^e_t(z),z)=0.$$

By assumption $q^s_t(p,z) \geqq q^s_t(p,z')$, so that

$$e^s_t(p^e_t(z'),z)=e^s_t(p^e_t(z'),z')+q^s_t(p^e_t(z'),z)-q^s_t(p^e_t(z'),z') \geqq e^s_t(p^e_t(z'),z')=0.$$

Hence,

$$e^s_t(p^e_t(z'),z')-e^s_t(p^e_t(z'),z) \leqq 0.$$

Using a mean value theorem, with $\bar{p}$ between $p^e_t(z)$ and $p^e_t(z')$, we have

$$e^s_t(p^e_t(z'),z')-e^s_t(p^e_t(z'),z)=\frac{\partial e^s_t}{\partial p}(\bar{p},z')\cdot(p^e_t(z')-p^e_t(z)) \leqq 0.$$

The derivative of $e(p, z)$ with respect to p is

$$\frac{\partial e^s_t}{\partial p}(p, z) = \frac{\partial q^s_t}{\partial p}(p, z) - \frac{\partial q^d_t}{\partial p}(p) > 0,$$

implying that

$$p^e_t(z') \leqq p^e_t(z). \quad ||$$

Proof of Lemma 2.

First consider the ratio

$$\beta^*(x) = \frac{q^e(1|x) - q^e(0|x)}{p^e(1|x) - p^e(0|x)}.$$

By Theorem 1 this is equal to

$$\int_0^\infty E\left[\frac{\partial q^d_t}{\partial p}(p, z_t)\middle| p^e_t(z') \geqq p \geqq p^e_t(z), x_t = x\right] \cdot \omega^d(p|x)dp.$$

All we therefore have left to prove is that $\beta^*(x)$ does not depend on x and that it is equal to the ratio of reduced-form slope coefficients.

Consider $q^e(z|x)$ and $p^e(z|x)$. By Assumption 5 this is equal to

$$q^e(z|x) = q^e_0(z) + \gamma_q \cdot x,$$

and

$$p^e(z|x) = p^e_0(z) + \gamma_p \cdot x,$$

respectively. Substituting this in $\beta^*(x)$ gives

$$\beta^*(x) = \frac{q^e_0(1) - q^e_0(0)}{p^e_0(1) - p^e_0(0)},$$

which proves the claim that $\beta(x)$ does not vary with x.

Finally, consider again $q^e(z|x)$. With binary z, and using Assumption 5, this can be written as

$$E[q^e|z, x] = q^e(z|x) = q^e_0(z) + \gamma_q \cdot x = q^e_0(0) + z \cdot (q^e_0(1) - q^e_0(0)) + \gamma_q \cdot x,$$

demonstrating that the coefficient on z in this linear regression is equal to the difference $(q^e_0(1) - q^e_0(0)$. ||

Proof of Lemma 3.

By additivity of the residuals we can write the demand function as additive in the average demand function and the residual

$$q^d_t(p, z) = q^d(p, z|x_t) + \varepsilon^d_t,$$

for all z and p. By linearity in prices and instruments we can write the demand function as

$$q^d_t(p, z) = \beta_{t0} + \beta_{1t} \cdot p + \beta_{2t} \cdot z,$$

for all z and p. Combining these two assumptions implies that we can write the average demand function as

$$q^d(p, z|x) = \beta_0(x) + \beta_1(x) \cdot p + \beta_2(x) \cdot z.$$

Combining this with the assumption of linearity and additivity in covariates implies that $\beta_1(x)$ and $\beta_2(x)$ do not depend on x and we can write the average demand function as

$$q^d(p, z, x) = \beta_0 + \beta_1 \cdot p + \beta_2 \cdot z + \beta_3 \cdot x.$$

Hence the demand function is

$$q^d_t(p, z) = \beta_0 + \beta_1 \cdot p + \beta_2 \cdot z + \beta_3 \cdot x_t + \varepsilon^d_t.$$

At the observed prices and instruments we therefore have

$$q_t^e = \beta_0 + \beta_1 \cdot p_t^e + \beta_2 \cdot z_t + \beta_3 \cdot x_t + \varepsilon_t^d,$$

and similar for the supply function. ||

Proof of Lemma 4.

(i) Given Assumptions 6, 8, and 9, Lemma 3 implies that the demand and supply function can be written as

$$q_t^d(p, z) = \beta_0 + \beta_1 \cdot p + \beta_2 \cdot z + \beta_3 \cdot x_t + \varepsilon_t^d,$$

$$q_t^s(p, z) = \alpha_0 + \alpha_1 \cdot p + \alpha_2 \cdot z + \alpha_3 \cdot x_t + \varepsilon_t^s.$$

Conditional on x_t the only stochastic component of the demand and supply function is the pair of residuals $(\varepsilon_t^d, \varepsilon_t^s)$. Independence of the demand and supply functions of z_t is therefore equivalent to independence of the residuals and z_t, proving the first part of Lemma 4.

(ii) The difference $q_t^d(p, z) - q_t^d(p, z')$ is under these assumptions equal to $\alpha_2 \cdot (z - z')$, which can only be zero if $\alpha_2 = 0$. This proves the second part of Lemma 4.

(iii) The equilibrium price at $z = 0$ and $z = 1$ are respectively

$$p_t^e(0) = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + x_t \cdot \frac{\alpha_3 - \beta_3}{\beta_1 - \alpha_1} + \frac{\varepsilon_t^d - \varepsilon_t^s}{\beta_1 - \alpha_1},$$

$$p_t^e(1) = \frac{\alpha_0 - \beta_0}{\beta_1 - \alpha_1} + \frac{\alpha_2 - \beta_2}{\beta_1 - \alpha_1} + x_t \cdot \frac{\alpha_3 - \beta_3}{\beta_1 - \alpha_1} + \frac{\varepsilon_t^d - \varepsilon_t^s}{\beta_1 - \alpha_1}.$$

If these are to be different for any period it must be the case that $\alpha_2 - \beta_2$ differs from zero. Since Assumption 10 establishes that $\alpha_2 = 0$, it must be the case that β_2 differs from zero. This establishes the third part of Lemma 4.

(iv) The difference between $p_t^e(1)$ and $p_t^e(0)$ is $(\alpha_2 - \beta_2)/(\beta_1 - \alpha_1)$. It does not change sign from one period to the next because the coefficients are fixed. This proves the final part of Lemma 4. ||

Before proving Theorem 1, we state an additional lemma.

Lemma 5. *Suppose Assumptions* 1–4 *hold with* $p_t^e(z') \geqq p_t^e(z)$. *In addition let* $g_t(\cdot)$ *be any continuously differentiable function. Then*:

$$E[g_t(p_t^e(z')) - g_t(p_t^e(z)) | x_t = x]$$
$$= \int_0^\infty E\left[\frac{\partial g_t}{\partial p}(p) \middle| p_t^e(z) < p <_t^e(z'), x_t = x\right] \cdot \Pr(p_t^e(z) < p < p_t^e(x') | x_t = x) dp.$$

Proof of Lemma 5.

The expected difference $E[g_t(p_t^e(z')) - g_t(p_t^e(z)) | x_t = x]$ can be written as

$$E[g_t(p_t^e(z')) - g_t(p_t^e(z)) | x_t = x] = E\left[\int_{p_t^e(z)}^{p_t^e(z')} \frac{\partial g_t}{\partial p}(p) dp \middle| x_t = x\right]$$

$$= E\left[\int_0^\infty \frac{\partial g_t}{\partial p}(p) \cdot 1\{p_t^e(z) < p < p_t^e(z')\} dp \middle| x_t = x\right]$$

$$= \int_0^\infty E\left[\frac{\partial g_t}{\partial p}(p) \cdot 1\{p_t^e(z) < p < p_t^e(z')\} \middle| x_t = x\right] dp$$

$$= \int_0^\infty E\left[\frac{\partial g_t}{\partial p}(p) \middle| p_t^e(z) < p < p_t^e(z'), x_t = x\right]$$
$$\cdot \Pr(p_t^e(z) < p < p_t^e(z') | x_t = x) dp. \quad \|$$

Proof of Theorem 1.

First apply Lemma 5 with $g_t(p)=p$ to get

$$p^e(z',x)-p^e(z,x)=E[p_t^e(z')-p_t^e(z)\,|\,x_t=x]=\int_0^\infty \Pr(p_t^e(z)<p<p_t^e(z')\,|\,x_t=x)dp.$$

Second, apply Lemma 5 again with $g_t(p)=q_t^d(p)$ to get

$$\begin{aligned}&E[q_t^d(p_t^e(z'))-q_t^d(p_t^e(z))\,|\,x_t=x]\\&=\int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p)\,\middle|\,p_t^e(z)<p<p_t^e(z'),x_t=x\right]\cdot \Pr(p_t^e(z)<p<p_t^e(z')\,|\,x_t=x)dp.\end{aligned}$$

Finally, taking the ratio of these two differences gives

$$\begin{aligned}\frac{q^e(z'|x)-q^e(z|x)}{p^e(z'|x)-p^e(z|x)}&=\int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p)\,\middle|\,p_t^e(z')\geqq p\geqq p_t^e(z),x_t=x\right]\cdot\frac{\Pr(p_t^e(z)<p<p_t^e(z')\,|\,x_t=x)}{\int_0^\infty \Pr(p_t^e(z)<p<p_t^e(z')\,|\,x_t=x)dp}dp\\&=\int_0^\infty E\left[\frac{\partial q_t^d}{\partial p}(p)\,\middle|\,p_t^e(z')\geqq p\geqq p_t^e(z),x_t=x\right]\cdot\omega(p,z,z',x)dp.\quad ||\end{aligned}$$

Proof of Theorem 2.

Corresponding to each pair of instrument values (z^k,z^l) there is an instrumental variables estimand β^{z^k,z^l}. These $K(K+1)/2$ estimands are related through the equality

$$(p^e(v)-p^e(z))\cdot\beta^{v,z}=(p^e(v)-p^e(w))\cdot\beta^{v,w}+(p^e(w)-p^e(z))\cdot\beta^{w,z},$$

for all triples (v,w,z). Because

$$\beta^{z^k,z^l}=\frac{q^e(z^k)-q^e(z^l)}{p^e(z^k)-p^e(z^l)},$$

we can write

$$q^e(z^k)=q^e(z^0)+\beta^{z^k,z^l}\cdot(p^e(z^k)-p^e(z^0)),$$

and substituting sequentially, this leads to

$$q^e(z^k)=q^e(z^0)+\textstyle\sum_{m=1,k}\beta^{z^m,z^{m-1}}\cdot(p^e(z^m)-p^e(z^{m-1})).$$

The IV procedure estimates

$$\beta^g=\frac{\operatorname{Cov}(q_t^e,g(z_t))}{\operatorname{Cov}(p_t^e,g(z_t))}=\frac{E[q_t^e\cdot(g(z_t)-E[g(z_t)])}{E[p_t^e\cdot(g(z_t)-E[g(z_t)])}.$$

First consider the numerator:

$$\begin{aligned}E[q_t^e\cdot(g(z_t)-E[g(z_t)])]&=\textstyle\sum_{k=1}^K f_z(z^k)\cdot E[q_t^e|z_t=z^k]\cdot(g(z^k)-E[g(z_t)])\\&=\textstyle\sum_{k=0}^K f_z(z^k)\cdot E[q_t^e|z^0]\cdot(g(z^k)-E[g(z_t)])\\&\quad+\textstyle\sum_{k=1}^K f_z(z^k)\cdot\sum_{m=1}^k\beta^{z^{m-1},z^m}\cdot(p^e(z^m)-p^e(z^{m-1}))\cdot(g(z_k)-E[g(z_t)])\\&=\textstyle\sum_{m=1}^K\beta^{z^{m-1},z^m}\cdot(p^e(z^m)-p^e(z^{m-1}))\sum_{k=m}^K f_z(z^k)\cdot(g(z^k)-E[g(z_t)]).\end{aligned}$$

The factor multiplying β^{z^{m-1},z^m} is the numerator in λ_k. Similarly the denominator $E[p_t^e\cdot(g(z_t)-E[g(z_t)])]$ can be shown to be equal to the denominator in λ_k. The weights λ_k then clearly add up to one. ||

Proof of Theorem 3.

Take the limit of the two components of equation (3) separately. First:

$$\lim_{dz\downarrow 0}E\left[\frac{\partial q_t^d}{\partial p}(p,z_t)\,\middle|\,p_t^e(z+dz)\geqq p\geqq p_t^e(z)\right]=E\left[\frac{\partial q_t^d}{\partial p}(p,z_t)\,\middle|\,p_t^e(z)=p\right].$$

Second the weights:

$$\lim_{dz\downarrow 0}\omega(p)=\lim_{dz\downarrow 0}\frac{\Pr(p_t^e(z)\leqq p\leqq p_t^e(z+dz))}{\int_0^\infty \Pr(p_t^e(z)\leqq p\leqq p_t^e(z+dz))dp}=f_{p_t^e(z)}(p).$$

Then taking the ratio of the two gives the desired result. ||

Proof of Theorem 4.

By definition,

$$\beta(z)=\lim_{dz\downarrow 0}\frac{q^e(z+dz)-q^e(z)}{p^e(z+dz)-p^e(z)}=\frac{\partial q^e}{\partial z}(z)\Big/\frac{\partial p^e}{\partial z}(z),$$

and therefore we can write

$$q^e(z)-q^e(z_0)=\int_{z_0}^{z}\beta(y)\cdot\frac{\partial p^e}{\partial z}(y)dy.$$

Now

$$E[q^e(z_t)\cdot(g(z_t)-E[g(z_t)])]=0,$$

and hence

$$\begin{aligned}E[q^e(z_t)\cdot(g(z_t)-E[g(z_t)]) &= E[(q^e(z_t)-q^e(z_0))\cdot(g(z_t)-E[g(z_t)])\\ &=\int_{-\infty}^{\infty}\int_{-\infty}^{z}\beta(y)\frac{\partial p^e}{\partial z}(y)\cdot(g(z)-E[g(z_t)])f_z(z)dydz\\ &=\int_{-\infty}^{\infty}\beta(y)\cdot\frac{\partial p^e}{\partial z}(y)\int_{z}^{\infty}(g(z)-E[g(z_t)])f_z(z)dzdy.\end{aligned}$$

A similar calculation for $E[p^e(z_t)\cdot(g(z_t)-E[g(z_t)])$ completes the argument. ||

Proof of Corollary.

The corollary can be proven along the same lines as Theorem 1 with first applying Lemma 5 with $g_t(p)=\ln(p)$ and then applying the same lemma with $g_t(p)=\ln q_t^d(p)$. ||

Proof of Corollary 2.

The instrumental variables estimand is in this case without covariates equal to

$$\beta^*(x)=\frac{q^e(1)-q^e(0)}{p^e(1)-p^e(0)}.$$

Consider first the numerator. It can be written as

$$\begin{aligned}q^e(1)-q^e(0) &= E[q_t^e(1)-q_t^e(0)]=E[q_t^d(p_t^e(1))-q_t^d(p_t^e(1))]\\ &=E[\beta_{t0}+\beta_{t1}\cdot p_t^e(1)-(\beta_{t0}+\beta_{t1}\cdot p_t^e(0))]\\ &=\beta_{t1}\cdot(p_t^e(1)-p_t^e(0)).\end{aligned}$$

By assumption the denominator is equal to $E[p_t^e(1)-p_t^e(0)]$. This proves the first equality.

The two equilibrium prices are equal to

$$p_t^e(0)=\frac{\alpha_{t0}-\beta_{t0}}{\beta_{t1}-\alpha_{t1}},$$

$$p_t^e(1)=\frac{\alpha_{t0}-\beta_{t0}}{\beta_{t1}-\alpha_{t1}}+\frac{\alpha_{t2}}{\beta_{t1}-\alpha_{t1}},$$

and their difference is

$$p_t^e(1) - p_t^e(0) = \frac{\alpha_{t2}}{\alpha_{t1} - \beta_{t1}},$$

so that

$$\beta^* = \frac{E[\beta_{t1} \cdot (p_t^e(1) - p_t^e(0))]}{E[p_t^e(1) - p_t^e(0)]} = E\beta_{t1} + \frac{\operatorname{Cov}(\beta_{t1}, p_t^e(1) - p_t^e(0))}{E[p_t^e(1) - p_t^e(0)]} = E[\beta_{1t}] + \frac{\operatorname{Cov}(\beta_{1t}, -\alpha_{2t}/(\alpha_{1t} - \beta_{1t}))}{E[-\alpha_{2t}/(\alpha_{1t} - \beta_{1t})]}. \quad ||$$

Proof of Corollary 3.

If the demand function is additive in a function of price and a period specific component the derivative with respect to the price is, $\partial q^d/\partial p(p)$, common to all periods. The only averaging is then over the periods with the equilibrium prices bracketing p, leading to the IV estimand

$$\beta^* = \int_0^\infty \frac{\partial q^d}{\partial p}(p) \cdot \frac{\Pr(p_t^e(0) < p < p_t^e(1))}{\int_0^\infty \Pr(p_t^e(0) < r < p_t^e(1))dr}. \quad ||$$

Proof of Corollary 4.

This follows directly from Theorem 1, applied at each pair of values of the instrument. ||

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